Comparing the Effects of Different Timings to Build Computational and Procedural Fluency with Complex Computations

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ABSTRACT: An alternating treatments design was used to compare (a) three, one-minute timings plus feedback after each timing, (b) one, three-minute timing plus feedback, and (c) one, one-minute timing without feedback (no treatment) on the calculation rates of four seventh graders practicing three distinct mathematics complex computations. Complex computations included order of operations, adding and subtracting fractions with uncommon denominators, and long division with and without a remainder. Components of the intervention comprised of cue cards, practice sheets, and answer keys to self-manage feedback. Despite gains in correct problems per minute, performance differences could not be attributed to the number and length of timed trials. Student responding increased in relation to the most stable and predictable procedures. Future directions for research are shared.

KEY WORDS: Fluency Building, Mathematics Fluency, Complex Computation, Feedback, Self-Managed Interventions

THE PROBLEM

Students in the United States who enter school with deficits in mathematics typically continue to struggle or fail to reach benchmarks required to function proficiently in high school algebra

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(Duncan et al., 2007; National Mathematics Advisory Panel, NMAP, 2008). Evidence indicates the sharpest decline in mathematics performance occurs at the middle school level. The National Assessment of Educational Progress (2015) reports only 40% of grade four students performed at or above proficient. Forty-two percent scored at basic and 18% below basic. In grade eight, only 33% scored at or above proficient with 38% basic and 29% below basic. The performance of students with disabilities calls for further concern. By grade eight 24% scored basic and 68% below basic. Regrettably, students who fail to meet mathematics standards have a greater likelihood to fail courses, endure retention, and dropout (Calhoon, Emerson, Flores, & Houchins, 2007; Duncan et al., 2007; NMAP, 2008).

Although a variety of reasons account for poor performance in mathematics, a lack of computational and procedural fluencies play a substantial role (Calhoon et al., 2007; Geary, 2004; NMAP, 2008). For instance, students who grapple with math facts tend to work more slowly, inaccurately, and exhibit difficulties keeping up with pace of instruction (Biancarosa & Shanley, 2016; Clarke, Nelson, & Shanley, 2016; Geary, 2004; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Lin & Kubina, 2005). Overreliance on finger-counting, counting-up, or making tally marks to compute math facts divert cognitive resources (e.g., working memory) from mastering steps that lead to procedural fluency with complex computation (Geary, 2011; Gersten, Jordan, & Flojo, 2005; LeFevre, DeStefano, Coleman, 2005; Raghubar, Barnes, & Hecht, 2010). In turn, nonfluency has also shown to negatively impact conceptual understanding, new problem-solving approaches, and generalization (Fuchs et al., 2008; Geary, 2011; Gersten & Chard, 1999) further impacting critical skills such as estimation, word problem solving, proportional reasoning, and algebraic reasoning (Bryant, Bryant, Gersten, Scammacca, & Chavez, 2008; Dowker; 2003; Hecht, Close, & Santisi, 2003).

The National Common Core State Standards for Mathematics (Common Core State Standards Initiative, CCSS, 2010) have addressed the importance of fluency by establishing a sequence of standards starting in kindergarten with whole numbers and extending through grade seven with fundamental algebraic equations. When students meet fluency standards, they typically *retain* and then *apply* the skill(s) in more advanced topics (Binder, 1996; Johnson & Layng, 1996; Kubina & Morrison, 2000; Kubina & Yurich, 2012). However, when a breakdown in the sequence occurs, difficulties typically compound and decrease the future probability of a student successfully engaging the mathematics curriculum.

Despite national initiatives, standards, and research establishing the significance of mathematical fluency, the quality of practice that occurs in many classrooms fall short in promoting fluency (Daly, Martens, Barnett, Witt, & Olson, 2007; NMAP, 2008; Riccomini & Witzel, 2010). In a study completed for NMAP (2008), algebra teachers (n = 748) cited the need for student fluency in basic skills such as fractions and decimals, order of operations, and positive and negative integers. Teachers also preferred that students use internalized cognitive problem-solving strategies rather than rely on calculators (Hoffer, Venkataraman, Hedberg, & Shagle, 2007). Researchers suggest a lack of attention to instruction that builds fluency and automaticity is evident in textbooks (NMAP, 2008; Witzel & Riccomini, 2007), and may reflect the pedagogical philosophies of textbook writers (Polikoff, 2015; Powell, Fuchs, & Fuchs, 2014).

PREVIOUS RESEARCH

To learn a new concept or skill students first concentrate on acquisition and conceptual understanding (Archer & Hughes, 2011; Ardoin & Daly, 2007; Binder, 2003; Haring & Eaton, 1978; National Council of Teachers of Mathematics, 2014). Students subsequently engage in systematic practice to reach fluency. Fluency refers to a skill performed to a high level of accuracy plus speed, reflected in a competent performance (Binder, 1996). Automatic execution of smaller, element skills saves cognitive resources that the learner can use when performing more complex skills (e.g., math facts to long division) (Raghubar et al., 2010; Sweller, Ayres, & Kalyuga, 2011). Systematic practice to build fluency *does not* suggest sacrificing conceptual understanding; fluency operates in tandem with acquisition and conceptual understanding to successfully transition to more difficult topics (Biancarosa & Shanley, 2016; NMAP, 2008; Partnership for Assessment of Readiness for College and Careers, PARCC, 2014).

Unfortunately, a paucity of research exists on how to build fluency with complex computations. Intervention research for mathematics fluency primarily focuses on simple computation (Foegen, Olson, & Impecoven-Lind, 2008; Geary et al., 2007). In a review of simple computation fluency interventions, Codding, Burns, and Lukito (2011) reported that instruction incorporating modeling and systematic practice on three or more components yielded the largest effect sizes.

The three components include reviewing the problem and solution, receiving immediate feedback, and participating in an error correcting procedure that reinforces correct responding versus errors (Burns, VanDerHeyden, & Boice, 2008, Daly et al., 2007; Fuchs et al., 2008). Practice without a modeling component generated the smallest effect size, which indicates that interventions should provide focus on individual problems and responses (Codding et al., 2011). Self-managed or student directed interventions also yielded significant effect sizes, highlighting improved focus, incentive, and responsibility over learning with minimal teacher mediation (Codding et al., 2011; Hughes, Korinek, & Gorman, 1991; Mace, Belfiore, & Hutchinson, 2001; McDougall & Brady, 1998; Reid, Trout, & Schartz, 2005).

THE SOLUTION

To develop computational fluency, the research literature supports a sequence of timed trials whereby trials are followed by immediate feedback (Brady & Kubina, 2010; Bullara, Kimball, & Cooper, 1993; Chiesa & Robertson, 2000; Kubina & Yurich, 2012; Miller, Hall, & Heward, 1995; Stromgren, Berg-Mortensen, & Tangen, 2014). Timed trials elevate the number of opportunities to respond while immediate feedback encourages correct responding versus errors (Mace et al., 2001; Reid et al., 2005; Stocker & Kubina, 2017). Researchers and teachers typically measure responses via digits correct per minute (DCPM) or correct problems per minute (CPPM) to score each performance (Johnston & Pennypacker, 2009).

When a student executes an element skill fluently, both the student and educator can have more confidence progressing to more complex skills. Self-managed feedback not only reinforces independent learning and increases motivation (Burns, Codding, Boice, & Lukito, 2010; Hattie & Timperley, 2007), but reduces a large amount of time teachers would otherwise spend providing individual feedback to large groups of students. Self-managed, fluency building activities with complex computation have the capacity to make mathematics instruction more efficient.

Present Investigation

Middle school students rely on the fluent execution of smaller, element skills (e.g., math facts, multi-digit computation) learned in previous grades to solve more complex problems. Students who successfully transition between skills have a distinct advantage over non-fluent students later in the high school algebra curriculum (NMAP, 2008). Order of operations, long division, and adding and subtracting fractions with unlike denominators serve as examples of complex computations that require fluency (CCSS, 2010). A lack of evidence-based research exists on self-managed interventions designed to increase fluency with complex computations. Instructional concerns such as applying appropriate timing(s), use of answer keys, and scaffolds that outline steps to the procedure warrant the present investigation.

To examine the effects of fluency building with self-managed feedback for complex computation, the experimenter posed the following questions: What effect does fluency building with a self-managed feedback component have on student performance with order of operations, long division with and without remainders, and adding and subtracting fractions with unlike denominators? Also, what performance differences occur in students between a three, one-minute fluency building intervention, a one, three-minute fluency building intervention, and a baseline condition?

EVIDENCE OF EFFECTIVENESS

Participants and Setting

A seventh-grade mathematics teacher nominated four students experiencing difficulties executing complex computations fluently and secured parental consent. Two female students (Cara and Poppy) and two male students (John and Jono) participated in the study. All four students had received instruction in the skills examined in the present investigation. Located in a Pennsylvania charter school, the intervention took place in a separate room next to the main office where small group instruction and meetings occur. The room had a long conference table where the four students and experimenter sat in the same seats for the 15 days of intervention.

Independent Variables

Two independent variables were applied in the study; each representing a different timed variation of the same fluency building intervention. The first independent variable or fluency building condition included three, one-minute timed trials. Three practice sheets replicated the same set of problems to reinforce correct responses. Students were encouraged to advance further and "beat their previous score" on the next timed trial. Following each one-minute timed trial, the students self-managed feedback using an answer key for 30 seconds. The second independent variable included one, three-minute timed practice trial. After three-minutes elapsed, the students again self-managed feedback using an answer key for 90 seconds. During both fluency building conditions, the students had access to a cue card that outlined the steps of the corresponding algorithm.

Experimental Design

Design

An adapted alternating treatments design was selected to compare and evaluate the effects of the independent variables (i.e., fluency building) on student performance (Cooper, Heron, & Heward, 2007; Johnston & Pennypacker, 2009; Kazdin, 2011; Sindelar, Rosenberg, & Wilson, 1985). The adapted alternating treatments design entailed creating equal sets of instructional items to be taught using different methods. Each set was equally difficult to learn, randomly assigned, and alternated (Sindelar et al., 1985).

The three conditions were systematically alternated each day to isolate the influence of the intervention assigned to the different conditions (Cooper et al., 2007; Kazdin, 2011; Sindelar et al., 1985). The lead researcher randomly assigned the three skills to different intervention conditions for each student (see Table 1) and counterbalanced the order in which the students received the three conditions (see Table 2). Randomly assigning the three different skills to three different conditions addresses confounds that could occur when students share the same skill and condition. Alternating the order of conditions controls unwanted effects of a static order that may favor one condition. While counterbalancing the order of fluency building

Table 1. Intervention Assignments

Student	*Baseline*	*Intervention #1*	*Intervention #2*
Cara	Order of Operations	Add/Sub Fractions	Long Division
John	Long Division	Order of Operations	Add/Sub Fractions
Jono	Add/Sub Fractions	Order of Operations	Long Division
Poppy	Add/Sub Fractions	Long Division	Order of Operations

Baseline: no practice; Intervention 1: Three, one-minute practice trials; Intervention 2: One, three-minute practice trial.

Table 2. Daily Alternated Schedule

Day			
1	Baseline	Intervention #1	Intervention #2
2	Intervention #2	Baseline	Intervention #1
3	Intervention #1	Intervention #2	Baseline
4	Repeat above sequence	Repeat above sequence	Repeat above sequence

Baseline: no practice; Intervention 1: Three, one-minute practice trials; Intervention 2: One, three-minute practice trial

conditions may control sequential confounding, carryover or practice effects can impact student performance indicating a critical problem to validity (Sindelar et al., 1985).

Dependent Variable

The dependent variable consisted of number of CPPM. Students were assessed after each condition for a total of three, one-minute assessments per day for the (a) one minute, control condition, (b) three, one-minute practice condition plus feedback, and (c) one, three-minute practice condition plus feedback. Each assessment contained more problems than a student could complete. This eliminated the chance of placing an artificial ceiling on performance.

Materials

Student materials consisted of (a) daily practice sheets, (b) answer keys for feedback, (c) cue cards that outline steps to solve the corresponding skill, and (d) daily assessments. Experimenter materials included (a) instructions, (b) procedural integrity checklists, (c) stopwatch, and (d) an intervention schedule. Occasionally, video was taken to evaluate procedures and assess student performance.

Three exclusive sets of practice sheets, corresponding answer keys, and assessments focused on either order of operations, long division with and without remainders, or adding or subtracting fractions with unlike denominators. Each practice sheet and assessment included nine problems. Below lists the decision rules for each complex computation to balance level of difficulty between assessments.

Order of Operations:

- eighteen sets of parentheses total, two per problem;
- nine exponents total, 1 per problem with products of 27 or less;

- five to eight multiplication facts per assessment, no more than two per problem;
- five to eight division facts per assessment, no more than one per problem;
- five to eight addition facts per assessment, no more than two per problem;
- five to eight subtraction facts per assessment, no more than two per problem

Long Division w/ and w/o Remainders

- the nine problems have one-digit divisors, two to nine divisors randomly assigned
- one problem with two-digit dividend
- three to four problems with three-digit dividend
- three to four problems with four-digit dividend
- four to five problems with remainders counterbalanced

Adding or Subtracting w/ Unlike Denominators

- common denominators occur between 4 and 81
- five problems have denominators with products up to 35.
- four problems have denominators with products up to 81
- addition and subtraction of fractions counterbalanced
- four problems counterbalanced simplifying and/or converting improper fractions

Procedure

Pre- and Post-Simple Computation Assessments

Before the start of the fluency building intervention, the students completed two, one-minute simple computation assessments—one for multiplication and one for division. The students then completed two more one-minute assessments the day after fluency building intervention ended to evaluate effects of fluency building with complex computation had on simple computation.

Fluency Building

Packages of fluency building practice sheets were placed (e.g., cue cards, practice sheets, answer keys, assessments, and cue cards) on a long rectangular table. Students chose permanent seats for the duration of the experiment and listened to the first of four sets of instructions corresponding with the intervention schedule. The instructions requested the students to (a) show all their work, (b) work left to right across the page starting with problem number one, (b) not skip problems, and (d) complete the task as rapidly as possible. The instructions also requested students to (a) calculate the remainder to the tenths or one decimal place and (b) remember to simply fractions and/or convert to a mixed number. During the assessment, the experimenter prompted a student to "please continue working" when he or she paused for more than five seconds, had a question, or caused a disruption before the timer expired.

On the first day, students started with the no-treatment condition. The students completed the one-minute timed trial, tore off the paper, and handed it to the researcher. The students then attended to the first of three, one-minute practice sheets. Following this first timed fluency building exercise, the students tore off the practice sheet and evaluated their work from an answer key (the next page) for 30 seconds. The students then tore off the answer key, handed in the first practice sheet, turned over the answer key, and then repeated the same process two more times. The three, one-minute fluency building trials produced a total three minutes of fluency building and 90 seconds of self-assessment and feedback. Next, the students completed a one-minute assessment for the dependent variable without the cue card.

For the second fluency building condition, the students practiced for three minutes with a cue card and then self-evaluated their work for 90 seconds with the answer key. After feedback, the student completed another one-minute assessment for the dependent variable. Afterward the experimenter thanked the students for their participation and hard work. The experimenter then promptly collected, scored, and inputted the data into a spreadsheet for evaluation. Each student participated for a total of 15 intervention days.

Procedural Integrity

A procedural integrity checklist ensured accuracy and consistency in implementation of the intervention by confirming the readiness of practice sheets, assessments, answer keys, cue cards, and instructions for administering the fluency building intervention. On four separate days, a research assistant checked procedural integrity. Training consisted of reviewing the materials and participating in a simulated procedural integrity check. Computing procedural integrity consisted of dividing the number of steps correctly executed over the total number of possible steps, then multiplying by 100 (Johnston & Pennypacker, 2009; Kazdin, 2011). The mean procedural integrity came to 100%.

Accuracy

Accuracy signifies the quality to which experimental values deliver a precise account of behavior that transpired during an experiment. Accuracy delivers more information than inter-observer agreement by calculating the exact values of experimental data (Johnston & Pennypacker, 2009; Kostewicz, King, Datchuk, Brennan, & Casey, 2016). In the present experiment, the lead investigator created an answer key for the assessments. The lead investigator and research assistant corrected written student responses against the answer key. The answer key used by the lead investigator and research assistant served as the true value or 100% agreement.

Retention

Approximately 30 days after the last day of fluency building, the students took three, one-minute assessments—one for each skill area to measure retention. Retention refers to long-term maintenance or keeping a skill in memory in the absence of practice (Kubina & Yurich, 2012).

Data Display and Analysis

Cumulative line graphs were employed to display CPPM across no treatment and the two intervention conditions. Cumulative graphs are additive each score represents an accumulated total of CPPM from all previous days (Kazdin, 2011). When comparing performance between conditions, a steeper slope represents a higher response rate (Cooper, Heron, & Heward, 2007). Bar graphs were employed to record the change in weekly median number of CPPM. Bar graphs offer a simple and efficient summary of the data but sacrifice showing trend and variability in response rates (Cooper et al., 2007). For the purpose of this analysis, cross-referencing the cumulative CPPM from the daily assessments on line graphs and the weekly median of CPPM from the bar graphs provide a snapshot of student performance.

RESULTS

Figures 1–4 display the daily data recorded for CPPM and the complex computations for each participant. Figure 5 displays the weekly median for CPPM and the complex computations for each participant. Table 3 contains all pre and post assessment scores for simple computation. All four participants showed improvement following the fluency building intervention except for Jono solving order of operations. All four participants showed improvement solving for long division regardless of the treatment condition as indicated by the weekly median scores. The following results provide an analysis on student performance accompanied by *italicized* element skills that represent examples students had difficulty executing successfully.

Cara. During Week 1 of the no-treatment condition (order of operations), Cara completed 7 correct problems on assessments and produced a median of 1 CPPM. She accumulated 14 correct problems by the end of Day 10 but remained at a median of 1 CPPM for Week 2. Cara accrued 23 successful correct problems at the end of Day 15 and showed a slight increase in weekly median to 2 CPPM. She produced *sporadic errors in computation* unrelated to any specific element skill and did not commit an error over the last four days. She reached 3 CPPM on the last day. On the retention measure, Cara completed 3 CPPM and 0 IPPM.

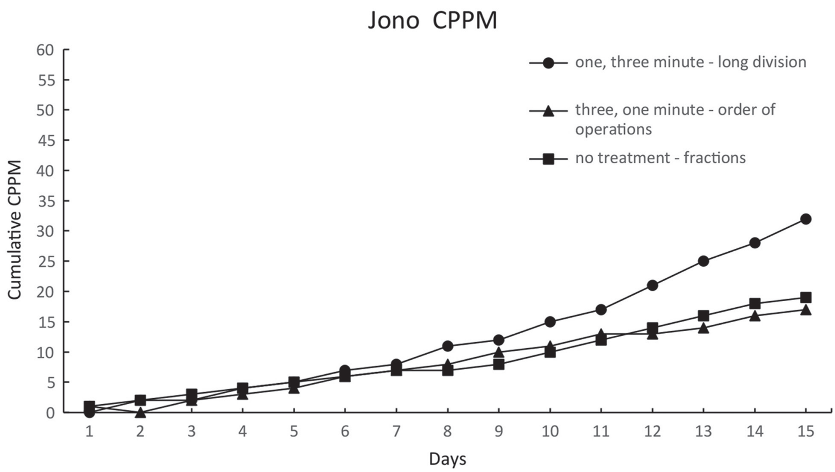

Figure 1. Jono.

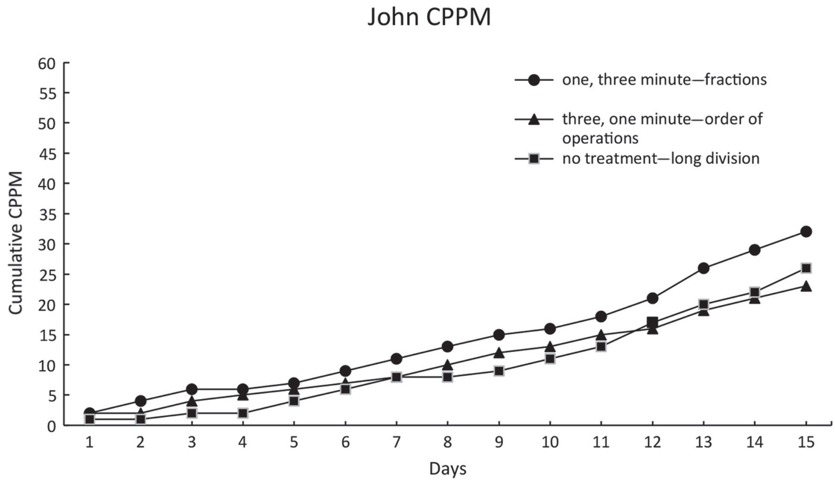

Figure 2. John.

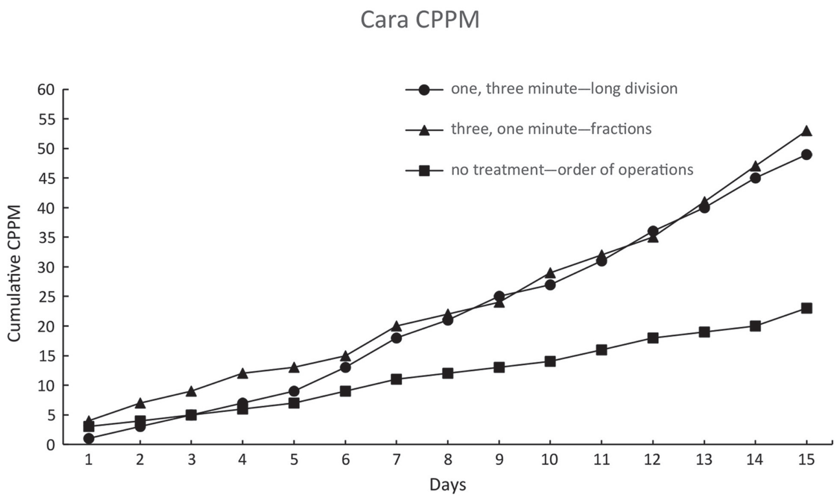

Figure 3. Cara.

By the end of Week 1 of the three, one-minute fluency building condition (fractions), Cara produced 13 correct problems on assessments and yielded a median of 3 CPPM. She accumulated 29 correct problems by the end of Week 2 and produced a median of 5 CPPM. Cara accrued 53 correct problems on assessments at the end of Week

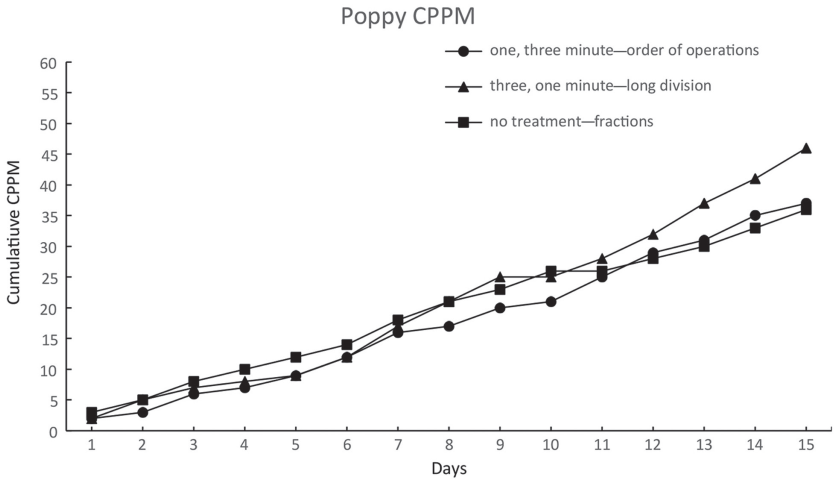

Figure 4 Poppy.

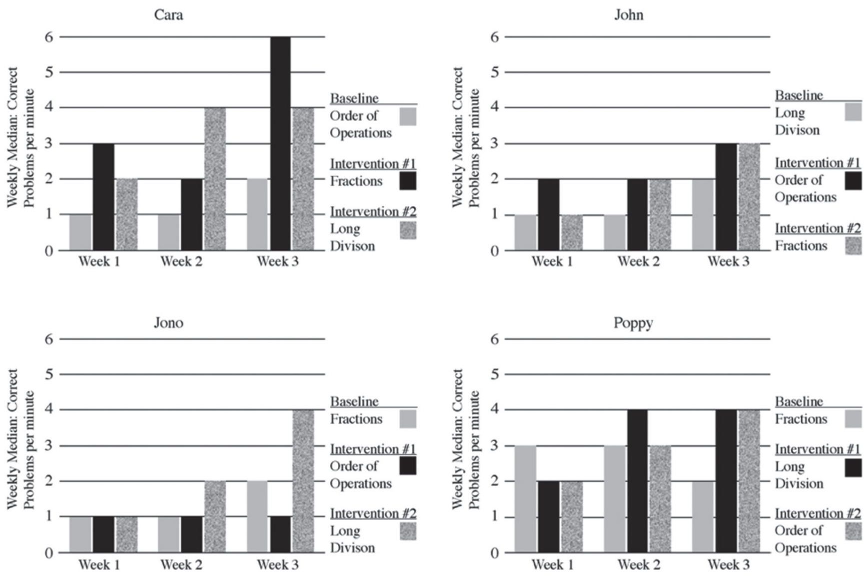

Figure 5. Weekly Median Correct Problems per Minute.

3 with a median of 6 CPPM. She produced only one IPPM on three separate days by not *changing improper fractions to mixed numbers* or *changing fractions to lowest terms*. Cara performed at a similar level

Table 3. Results from Simple Computation Probes

Name	Operation	Initial Probe	Exit Probe	± DCPM Change
Cara	Multiplication	42 DCPM	39 DCPM	–3 DCPM
	Division	13 DCPM	18 DCPM	+5 DCPM
John	Multiplication	21 DCPM	24 DCPM	+3 DCPM
	Division	12 DCPM	18 DCPM	+6 DCPM
Jono	Multiplication	23 DCPM	35 DCPM	+12 DCPM
	Division	16 DCPM	19 DCPM	+3 DCPM
Poppy	Multiplication	23 DCPM	49 DCPM	+26 DCPM
	Division	17 DCPM	30 DCPM	+13 DCPM

successfully completing five CPPM and committing zero IPPM on the retention measure.

Cara produced a total of 9 correct problems on the assessments and a median of 2 CPPM for Week 1 in the three, one-minute fluency building condition (long division). She accumulated 27 correct problems on assessments by the end of Day 10 with a median of 4 CPPM for Week 2. Cara accrued 49 correct problems on assessments by the end of Week 3 yielding a similar 4 CPPM. Cara produced 1 IPPM on two occasions exhibiting *difficulties when computing remainders*. On the retention measure, she yielded 4 CPPM and 1 IPPM from attempting to solve the problem "in her head." Cara showed her work on the remaining problems. Her results from the simple computation probes showed a slight decrease in multiplication from 42 DCPM to 39 DCPM and an increase by five DCPM with division facts from 13 to 18.

John. For Week 1 of the no-treatment condition (long division), John completed 4 correct problems on assessments and produced a median score of 1 CPPM. He accumulated 11 correct problems by the end of Day 10 and yielded a similar Week 2 median score of 1 CPPM. By Day 15, John accrued 26 correct problems on assessments and increased to a Week 3 median score of 2 CPPM. Like Cara, he had trouble *computing remainders*, but corrected the element skill by the last week of the study. John completed 3 CPPM and 1 IPPM on the retention measure.

Week 1 of the three, one-minute fluency building condition (order of operations) saw John yield a total of 6 correct problems on the assessments and a median score of 2 CPPM. By Day 10, he accumulated 13 correct problems on assessments and produced a Week 2 median score of 2 CPPM. John completed a total of 23 correct problems on assessments by Day 15 and increased his Week 3 median score to 3 CPPM. John's IPPM stemmed from inconsistent *computation* *with decimals, and positive and negative numbers*. He only produced 1 CPPM on the retention measure.

Over Week 1 of the one, three-minute condition (add/sub fractions), John produced 6 correct problems on assessments and a median score of 1 CPPM. He accumulated 16 correct problems by Day 10 on assessments and yielded a Week 2 median score of 2 CPPM. After 15 days of intervention, John successfully completed 32 problems and increased his Week 3 median score to 3 CPPM. John did not emit an IPPM over the span of the study with fractions. He scored four CPPM and zero IPPM on the retention measure. His results from the simple computation probes showed a slight increase in multiplication from 21 DCPM to 24 DCPM and an increase in division facts from 12 to 18.

Jono. During the first week of the no-treatment condition (add/sub fractions), Jono completed a total of 5 correct problems on the assessments and yielded a median score of 1 CPPM. He accumulated 10 correct problems by Day 10 and maintained a median score of 1 CPPM for Week 2. By the end of Day 15, he completed 19 correct problems and continued to produce a median score of 1 CPPM for Week 3. Jono often *applied the wrong operator (i.e., +, –, x, ÷)* leading to inaccurate responses. He scored 1 CPPM on the retention measure.

For the first week of the three, one-minute fluency building condition (order of operations), Jono successfully completed a total of 4 correct problems and yielded a median of 1 CPPM. By Day 10, he accrued 11 correct problems and continued to produce a Week 2 median of 1 CPPM. The end of Day 15 saw Jono accumulate a total of 17 correct problems and continue with a Week 3 median of 1 CPPM. Similar to adding and subtracting fractions in the no-treatment condition, Jono applied the *wrong operator (i.e., +, –, x, ÷)* which hindered his performance. He also performed 1 CPPM on the retention measure.

In the one, three-minute fluency building condition (long division), Jono had the most success. During Week 1, he successfully completed 5 correct problems and yielded a median of 1 CPPM. By Day 10, Jono accumulated 15 correct problems and established a Week 2 median of 2 CPPM. He accrued 32 correct problems by Day 15 and posted a Week 3 median of 4 CPPM. His typical error pattern occurred miscalculating the first step of the recurring procedure when *dividing the divisor into the appropriate number(s)of the dividend*. Jono successfully answered 4 CPPM and emitted 0 IPPM on the retention measure. His results from the simple computation probes showed an increase in multiplication from 23 DCPM to 35 DCPM for a robust gain of 12 DCPM and a smaller increase in division from 16 DCPM to 19 DCPM.

Poppy. In the no-treatment condition (add/sub fractions), Poppy completed a total of 12 correct problems and registered a Week 1 median of 3 CPPM. By Day 10, she accumulated 24 correct problems and maintained the same median as Week 1 with 3 CPPM. Poppy accrued 37 correct problems by Day 15, but her Week 3 median dropped to 2 CPPM. Poppy had difficulty remembering to *change improper fractions to mixed numbers*—a skill she executed sporadically leading to the one IPPM three separate days. On Day 11, she did not produce a correct problem and made sporadic computation errors. Interestingly, Poppy yielded 4 CPPM and 0 IPPM on the retention measure.

Through the first week of the three, one-minute fluency building condition (long division), Poppy successfully emitted a total of 10 correct problems and yielded a Week 1 median of 2 CPPM. By Day 10, she accrued 25 correct problems and continued to produce a Week 2 median of 4 CPPM. Poppy accumulated a total of 46 correct problems and a Week 3 median of 4 CPPM. Like Cara and John in the study, her IPPM originated from *difficulty computing remainders*. By the last week, Poppy self-corrected the consistent error pattern with computing remainders and did not commit an IPPM. She produced 5 CPPM and 0 IPPM on the retention measure.

Poppy exhibited a sporadic performance in the one, three-minute fluency building condition (order of operations). She yielded between one and four CPPM over the first two weeks of intervention. Week 1 median was 2 CPPM and Week 2 was 3 CPPM. During the last week of fluency building, Poppy produced between three and five CPPM. She committed one IPPM on four separate days and two IPPM on one occasion. Like John, her inconsistencies in performance derived from inaccurately computing *decimals, and positive and negative numbers*. Poppy produced 4 CPPM and 0 IPPM on the retention measure. She showed a robust increase for both simple computation assessments gaining 23 DCPM and 13 DCPM in multiplication and division, respectively.

DISCUSSION

The first experimental question posed was, "What effects does fluency building with a self-managed feedback component have on student performance with order of operations, long division with and without remainders, and adding and subtracting fractions with unlike denominators?" After 15 days of fluency building the data indicate students had the most success with long division followed by addition and subtraction with fractions. A couple of reasons can plausibly explain student success with long division. First, long division had the most stable, compact, and predictable problem-solving procedure of the three skills which allowed students to work more efficiently. The participants repeated the same sequence between one and three times in each problem and then repeated the same procedure when engaging the next problem. The predictable and efficient problem-solving sequence executed at a brisk pace during fluency building with long division translated to increased momentum and response speed which in turn, increased the weekly median number of correct problems over the span of the investigation (Binder, 1996; Lee, 2006; Stocker & Kubina, 2017).

Second, long division problems contained only whole numbers. Negative integers, fractions, and decimals were not included in the dividend or divisor. The only unpredictable instance in problemsolving was when randomly calculating a decimal remainder which occurred at least twice during each timed practice trial or assessment. Cara occasionally committed 1 IPPM when computing remainders and Poppy between 1 and 3 IPPM. Jono had trouble dividing the divisor into the appropriate number(s) of the dividend suggesting he requires more practice solving simple computation with the division bracket. Yet, all three students rectified their skill deficits by Day 11. And by the end of the study, all three students in the fluency building condition reached a weekly median of 4 CPPM. This information suggests timed trials and immediate feedback via answer key after each timed trial led to enhanced speed, accuracy, and quality of the response (Binder, 1996; Hughes, Beverley & Whitehead, 2007; Kubina & Yurich, 2012, Stocker & Kubina, 2017).

Although less efficient and compact than long division, adding and subtracting fractions had a more predictable problem-solving procedure. As a result, the fluency building intervention still had a positive effect. For instance, in the no-treatment condition, Jono only increased 1 CPPM for the weekly median and Poppy decreased by 1 CPPM. In the fluency building conditions, Cara gained 3 CPPM for the median while John increased 2 CPPM and did not commit an IPPM. Cara and Poppy occasionally had difficulty completing the problem when having to simplify fractions and change improper fractions to mixed numbers, yet, Cara only committed 1 IPPM over the last five days of fluency building while Poppy committed 4 IPPM over the same time-period in the no-treatment condition. Jono also had 4

IPPM in the last week of intervention and yielded only 1 CPPM on the retention measure. The individual data provides initial evidence that students can also successfully self-managed feedback with complex computations during fluency building activities (Burns et al., 2008, 2010; Hattie & Timperley, 2007).

Conversely, the four students had the most difficulty keeping a brisk, steady pace with order to operations regardless of the fluency building condition. Due to the unpredictable problem-solving sequences and random operators embedded in order of operations, students worked slower and made less progress as indicated by the cumulative graphs and weekly median scores. For instance, John and Poppy hesitated computing decimals and positive and negative integers. Like his performance with fractions, Jono continued to confuse operators when computing. Hesitation and persistent error patterns that occur in complex problem-solving practice activities that do not respond to self-managed feedback necessitate remedial instruction for acquisition and separate fluency building activities to remedy the element skill deficits (Beverley, Hughes, & Hastings, 2009). Taking into consideration the abovementioned skill deficits and frustration and instruction levels with simple computation, order of operations should not have been a selected for the study.

The second experimental question asks, "What performance differences occur in students between a three, one-minute fluency building intervention, a one, three-minute fluency building intervention, and a baseline condition?" Researchers often test different iterations of interventions (e.g., structure, timings, feedback, assessment) to determine a "best-fit" for individual learners or groups of learners (e.g., Brady & Kubina, 2010; Poncy, Skinner, & Jaspers, 2007; Skinner, Bamberg, Smith, & Powell, 1993). For instance, Brady and Kubina (2010) measured performance differences between using three, 20 second timings versus one, 60 second timings when solving for simple computation. The present investigation could not determine an advantage of using three, one-minute practice trials versus one, three-minute practice.

Although an attempt was made to balance the difficulty level of each skillset as reflected in the decision rules, long division proved to be the easiest, followed by adding and subtracting fractions, and then order of operations. Because of the significant imbalance between order of operations and the other complex computations, recommendations for future research include pre-testing students for fluency on smaller, element skills before selecting complex computations and timing students who are already fluent with select complex computations to ensure similar completion rates.

The students took two, 60-second pre and post assessments for simple computation. The practice effects between complex computations increased student performance with simple computation by the end of the study; however, three of the four students remained in the frustration range for division facts. All four students are in the instructional range for multiplication. It is likely that the increase in math facts fluency carried-over from the complex computations stimulated the most growth with long division (Sindelar et al., 1985).

Implications for Practice

Research in mathematics shows that fluency with element skills can impact the fluent execution of more complex skills (Lin & Kubina, 2005; Lin, Kubina, & Shimamune, 2011; McDowell & Keenan, 2002). While all four students in the present investigation exhibited skill deficits in element skills (i.e., math facts) that impeded optimal progress, error analysis from assessments suggest the students experienced more difficulty increasing speed and accuracy with order of operations due to the larger number of different element skills (i.e., positive and negative numbers, exponents, decimals) presented in random sequences. Therefore, it is recommended that teachers introduce fluency building for complex computation with shorter timings to increase opportunities for feedback from answer keys, then gradually increase length of timings to complete more problems (e.g., 30 second increments to three minutes).

Evidence-based practice activities spanning from acquisition to fluency include careful planning in which teachers must match appropriate materials to the skill-level of the student (Burns et al., 2008; Cawley, Parmar, Foley, Salmon, & Roy, 2001). Likewise, self-managed interventions with scaffolds such as cue cards that outline procedures require careful planning and matching. For teachers seeking to build fluency with complex computation, students should have acquired the procedure. Cue cards function as a temporary scaffold to assist students in the beginning as they build speed. The four students did not rely on the cue cards past the first few days.

The present study did not allow for additional remediation from a teacher or researcher; the students could only depend on the answer key for feedback. Classroom activities do not require the same guidelines, and as with most learning activities, a self-managed intervention still involves teacher mediation when appropriate. Teachers would typically engage in an immediate analysis of error patterns that students did not recognize on answer keys, pinpoint instruction for remediation, and apply further practice to reach mastery on the element skill(s) (Ashlock, 2006; Kubina & Yurich, 2012).

Social Validity

All four students preferred the one, three-minute time allocation for fluency building. The students did not quit or exhibit frustration over the span of the investigation. The students liked the opportunity to check their work with the answer key but wanted the opportunity to complete the practice sheet. Working for three minutes without pause suggests the choice in skills, level of difficulty, and appropriately matched materials were reinforcing and encouraged endurance—a key feature of fluency in which a student exhibits the capacity to stay on task at a certain speed and level of accuracy over an extended time frame (Binder, 1996; Fabrizio & Moors, 2003; Kubina & Yurich, 2012). Comments made by students included, "it helps me in math (class)," "I like getting better with practice," "this can help me get better grades," and "I can solve problems faster in class." All the students could see themselves committing to 10 minutes of fluency building per day.

CONCLUSION

Fluency building successfully increased speed and accuracy with long division and adding and subtracting fractions. For teachers, five to ten minutes of fluency building per day can function as an effective and efficient method to remedy issues that impact student performance in mathematics. Students who have not acquired element skillsets to a level of accuracy should wait before engaging in fluency building with complex computations. Other benefits of fluency building include supporting pace of instruction by lessening the use of inferior computing strategies and improving cognitive capacity to improve conceptual understanding (Biancarosa & Shanley, 2016). Although further research is necessary to fine-tune fluency building with complex computation, teachers can still implement and adjust fluency building procedures as necessary to enhance mathematics practice. JEBPS

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IMPLEMENTATION GUIDELINES

A. Create three practice sheets and answer key with more problems than the student can solve in the timing selected for practice. Create a separate assessment.
B. Say to the students:

"We're going to take (1) 3-minute math probe. I want you to show your work and write your answers to each of the order of operations problems. Start with the first problem and then continue to the next problem.

"You have a cue card placed in front of you to help you if you forget how to solve the problem."

"On long division problems with remainders, go out only one decimal place."

"On fractions, please make sure to reduce fraction and/or change improper fraction to a mixed number."

"Work as quickly as you can on each problem"

"At the end of each probe you will have 90 seconds, a minute and a half, to compare your answers to the answer key"

"When I say 'BEGIN' you can work ACROSS the page. Then go onto the next row."

"Are there any questions? (Pause)"

"Begin"

****If a student stops working before the test is done, say to the student: "Keep doing the best work you can."

C. Have students check their work using the answer key
D. Repeat two more times
E. Administer assessment to monitor progress
F. Provide feedback where necessary
G. Repeat fluency building daily for optimal gains in performance.