Examining Application Relationships: Differences in Mathematical Elements and Compound Performance between American, Japanese, and Taiwanese Students

Fan-Yu Lin, Ph.D.
Assistant Professor of Special Education
Robert Morris University

Richard M. Kubina Jr., Ph.D.
Associate Professor of Special Education
Pennsylvania State University

Satoru Shimamune, Ph.D.
Professor of Psychology
Hosei University

Abstract: *The concept of application refers to the process of behavioral elements combining to form a behavioral compound, which is essential in any advanced learning skill. The purpose of this study was to examine how the concept of application related to multiplication performance measured by percent correct (accuracy) and correct responses per unit of time (frequency, a measure of fluency) among students from the United States, Taiwan, and Japan. A total of two hundred eighty-nine students participated in this study. A testable model of how element and compound skills relate in computation proficiency was proposed and analyzed. The results showed that although the majority of the students achieved high levels of accuracy in single digit and multi-digit multiplication problems, students from Taiwan and Japan were more fluent in their basic multiplication than those from the United States. As predicted, these students were also more fluent in multi-digit multiplication. The results confirm previous research that fluency in basic skills can serve as a powerful predictor for the concept of application.*

Application refers to the process of behavioral elements, sometimes referred to as component behaviors, combining to form a behavioral compound, also referred to as a composite behavior (Binder, 1996; Haughton, 1980). Figure 1 shows how element behaviors can combine to form a compound behavior. In the example, two discrete elements behaviors, (A) using finger on right hand to press a piano key and (B) reading the C note, may exist in a person's repertoire. When they combine the new behavior (AB) is a behavioral compound, seeing a C note on a piece of sheet music and pressing the correct key on the piano. Other behavioral compounds may have two, three or more elements.

Application, like any synthesis reaction in chemistry when the combination of two or more substances results in a compound, requires activation energy for the two behaviors to combine. The activation energy necessary in application appears in the form a frequency measure (Johnson & Pennypacker, 1993). This initial discovery came from Haughton (1972) who observed one student performing multiplication facts (i.e., multiplying 0 – 5 facts) at a frequency of 30 correct per minute and another answering facts below 20 correct per minute. The first student went on to successfully answer more complex math facts (i.e., mixed multiplication facts) while the second student failed to learn the more complex math facts. In other words the higher frequency permitted the application of the skill element multiplication of basic facts, to a skill compound, multiplication of mixed facts.

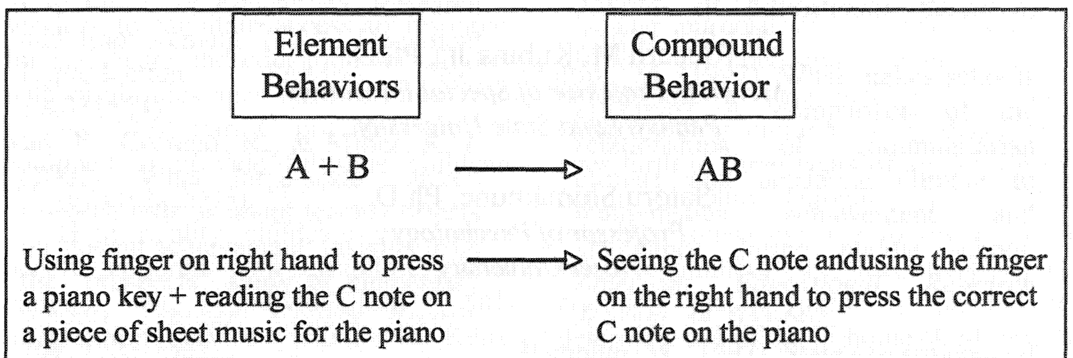

Figure 1. A Model for Application and Example of Two Behavioral Elements Combining To Form One Compound Behavior

Haughton's seminal observation of application coincides with the collision theory. With chemical reactions, reactants must collide to break their existing bonds and form new bonds resulting in a product. If too little energy exists, molecules will bounce off one another and not combine (Brown, LeMay, & Bursten, 1994). Haughton (1972) indicated that even though both students could perform the element behavior with some degree of accuracy, the math fact performance in the second student would not successfully apply to the compound behavior because it did not possess the necessary frequency.

Based on the application concept and a number of previous application studies (Berens, Boyce, Berens, Doney, & Kenzer, 2003; Bucklin, Dickinson, & Brethower, 2000; Chiesa & Robertson, 2000; Evans & Evans, 1985; Evans, Mercer, & Evans, 1983; Kubina, Young, & Kilwein, 2004; McDade, Rubenstein, & Olander, 1983; McDowell & Keenan, 2001; McDowell, & Keenan, 2002; McDowell, Keenan, & Kerr, 2002; McDowell, McIntyre, Bones, & Keenan, 2002; Smyth, & Keenan, 2002), Lin and Kubina (2005) hypothesized that a relationship should exist between elements and compound behaviors in mathematical behavior. In their study Lin and Kubina examined the frequency of one behavioral element, basic facts in multiplication (i.e., x 0 – 9 facts), and the compound behavior of complex multiplication facts (i.e., multiplication problems involving single, two-digit, and three-digit factors). After measuring the 1-minute frequencies of the 156 fifth grade students' single and multi-digit multiplication facts, a positive .745 correlation between the element and compound skill was found, which supports the essential role of basic skill frequencies in contributing to the learning in compound behaviors.

Additionally, when comparing the 5th grade sample against a performance standard for basic fact fluency (i.e., 80-120 correct digits per minute, Mercer, Mercer, & Evans, 1982), 14% of the sample met or exceeded the criterion. When contrasting the 5th grade students with the performance standard for multi-digit fact fluency (i.e., 40-60 correct digit per minute, Kubina & Lin, 2003) only 3% met or exceeded the criterion. Such dataindicate that like chemical reactions, with the minimum energy necessary to initiate a reaction varying from reaction to reaction (Brown, et al., 1994), behavioral elements differ in the requisite frequency for application to a compound behavior. In the case of the 156, 5^{th} grade sample, the majority of students did not possess the needed fluency, or activation energy, with the one behavioral element and therefore the low frequencies in the compound skill were expected.

If the frequency of behavioral elements in mathematical skills can be shown to reliably predict compound performance how teachers diagnose and resolve mathematical problems would be greatly enhanced. For example, a recent report by the National Mathematics Advisory Panel (2008) indicates computational proficiency is predicated on whole number fluency. Based on consensus reports such as the National Mathematics Advisory Panel, we suggest a testable model of how element and compound skills correlate in mathematics (Figure 3). Our model suggests that accuracy in many element skills for mathematics is essential in the initial acquisition and prerequisite to frequency building. However, it may not be sufficient to produce competency at the compound level.

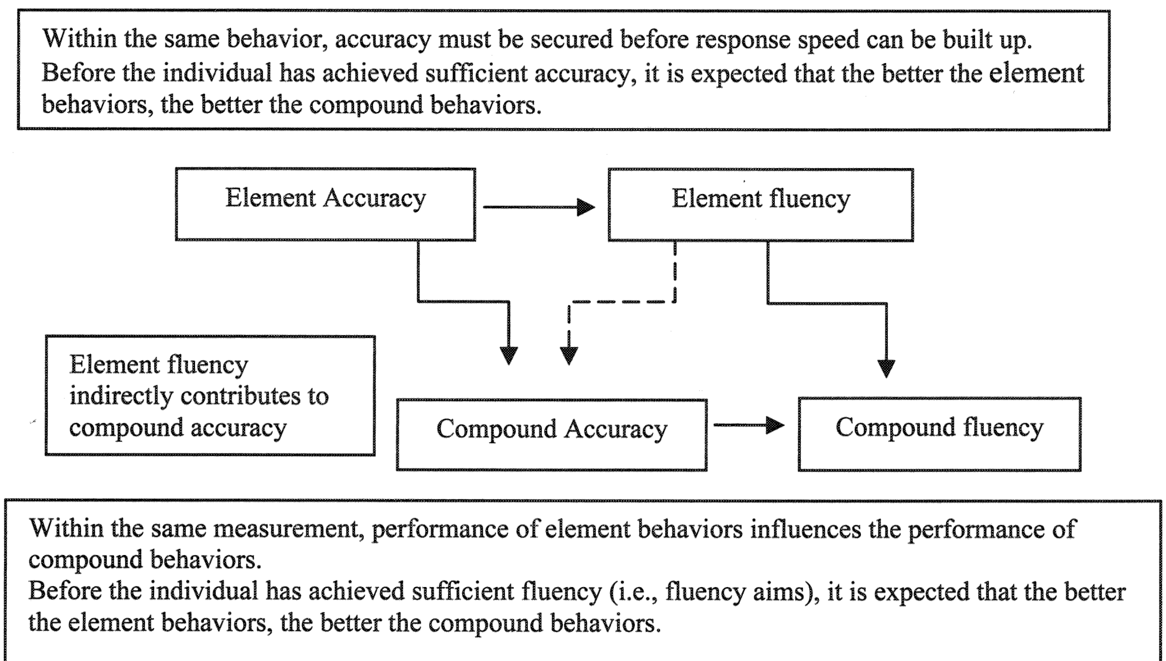

Figure 2. A Model for Relationships of Element Accuracy and Fluency, and Compound Accuracy and Fluency

While the Lin and Kubina (2005) study does provide evidence for the application model of elements directly effecting compound skills we seek to explore additional evidence as to the validity of our model. One area of research in mathematics that lends itself to investigation for the application model rests with large scale comparisons (e.g., Trends in International Mathematics and Science Study, 2004) and other research reports (e.g., Geary, Liu, Chen, Saults, & Hoard, 1999; US Department of Education, 2008) showing differences in basic and advanced computation among American and Asian students. We suggest these results can be attributed to application. In other words, Asian students may possess the requisite behavioral element frequencies necessary for the formation of a behavioral compound. If this hypothesis is true, we would expect Asian students to have higher frequencies in their element skills. Subsequently, we would also expect the performance gaps between American and Asian students would become larger with the increased complexity of skill, especially those compound skills dependent on skill elements. In the present study we attempted to study application, most notably in the relationships between skill elements and compounds, by asking the following questions:

1. How do fifth grade students from three different countries (i.e., United States, Japan, and Taiwan) differ in their performance with single-digit and multi-digit multiplications?
2. Do measures of accuracy and fluency distinguish student performance?
3. How is the element skill performance (single-digit multiplications) associated with the compound skill performance (multi-digit multiplications)?

Method

Participants

A total of 289 fifth grade students from Japan, Taiwan, and the United States participated in this study. The specific demographic details for the participating students in each country follow.

Japan - Two public elementary schools from a suburban school district in Tokushima prefecture, Japan participated in this study. From one school, a total of 55 students participated from two fifth-grade classrooms, and from the other school, 41 students came from one, fifth grade classroom. The school district from which the target schools were selected was located in a city which had a mixture of manufacturing industry, agriculture, and human-services. The city's average income was 4th in 24 cities in Tokushima prefecture. Although no formal data are available, socio-economic status of the students could be described as diverse; yet, the variability is relatively smaller, compared to most United States suburban public schools. None of the participating students attended learning support classes or received special education services. The total sample represented 96% of the fifth grade students served by the two schools.

Taiwan - Ninety-five students from three fifth-grade classes in one public elementary school in Taipei County participated. Located in the northern Taiwan, the region of Taipei County had the second highest household income average in Taiwan. The school was located in a diverse socio-economic urban community and the population in this region was highly concentrated. The average school size in Taipei County is much larger than most elementary schools in the United States. In the 2007/2008 academic year, there were approximately 2,500 students enrolled in the participating school, with a total of 14 fifth grade classes, more than 400 fifth graders. Among these participating students the teachers identified two students with learning disabilities. The total sample represented approximately 20% of the fifth grade students served by the participating school.

United States - Three public elementary schools from three school districts in central and southeastern Pennsylvania participated in the study. According to the Pennsylvania Department of Education, these schools were coded as urban and/or suburban communities. A total of 98 students participated. The total sample represented approximately 40% of the fifth graders served by the three schools. The principals reported approximately 10% of the participating students attended learning support classes or received special education services.

Materials

Students used test packets which had three sections. The first section, numeral writing, served two purposes: (1) providing an opportunity for students to practice time limited testing; and (2) serving as a reference for any near illegible digits in the actual computation sections. It had lined notebook paper to determine minimum competency skills by having students repeatedly write Arabic numerals from 0 to 9. The second section presented 156 random single-digit multiplication problems and assessed the students' element skill performances. The third section had 63 random multi-digit multiplication equations. The equations ranged from 2 or 3 digit multiplicands times 1 or 2 digit multipliers with an even distribution of problems with and without renaming/regrouping.

Due to the difference of conventionally used paper sizes, the experimenters in Japan and Taiwan printed the test packets on A4 paper (8.27" x 11.69") while the print margins used in United States were on standard letter size paper (8.5" x 11"). The experimenters provided copies of the assessment protocol upon request. Additionally, the study called for an electronic countdown timer to signal the beginning and ending times for the assessment intervals.

Procedure

All students first heard a brief description of the purpose of the study. Students were told they would answer each of the three test sections in one-minute as quickly and accurately as possible. The teachers/experimenters showed the students an electronic countdown timer and demonstrated the timing procedure before the one-minute assessment began. Before each one-minute assessment began, the experimenters prompted students for questions. After questions were answered, the teachers/experimenters began the timing by saying, "Please begin." The teachers/experimenters simultaneously started the countdown timer when they gave the verbal prompt.

Some slight differences in the testing administration and settings occurred across the three countries. In United States, participating students in each individual school came down as a group to an empty classroom or cafeteria. The experimenters, the first and second author, came in and conducted assessments. In Taiwan and Japan, the classroom teachers actually administered the timings following the scripts provided by the experimenters. Students remained in their designed classrooms when they took the tests.

Students were required to do two separate sections of multiplication, single-digit and multi-digit multiplication equations. The teachers/experimenters told the students to cross their answers out when they made a mistake instead of using erasers and to skip questions if they did not know how to complete the problem. The instructions of not focusing on errors and problems students could not complete helped reduce the likelihood that a resulting production of lower digits per minute accurately reflected present skill level. Each of the multiplication assessment sheets contained two pages. Students started writing answers from the first page of the section and continued working on the second page if they finished the first one before the timing ended.

Measurement Units

A correctly written digit served as the primary measurement unit for this study. The written response 35, for the equation

7 x 5 =, had a count of two correct digits. If a student wrote 34 for the previously mentioned equation she would receive a count of 1 correct and 1 incorrect for her written response (i.e., 3 in the tens column = correct, 4 in the ones column = incorrect). When students wrote algorithms for the multi-digit multiplication problems in the third section in addition to the final product each digit in an algorithm counted toward students' digits total. For example, the multiplication equation 261 x 52 = may have included an algorithm of 522 + 13050 equal to 13572. All of the digits combined yielded a total of 13 correct digits. If a student only wrote 522, 3 correct digits counted toward her total skill performance in this section even though the remaining digits and answer did not appear. Variation in algorithm writing affects the number of total correct digits. Students may write 0, or X, or leave it blank in the ones column on the second rows of the computation (i.e., 13050, or 1305_, or 1305x). To keep the measurement units consistent, one digit was automatically added to the total correct numbers if place value was clearly demonstrated. The teachers/experimenters reminded students that the assessment contained more questions than they could finish in the one-minute timing.

All completed test packets were compared to a preconstructed key. The total correct digits generated two variables — frequency and accuracy. The frequency variable comprised the number of correct digits written per minute. The accuracy variable was calculated by taking the percentage of correct digits from the total completed digits (sometimes represented as decimals in this study). Thus, this study analyzed four variables: element accuracy (i.e., percentage of correctly written digits for single-digit multiplication problems), element fluency (i.e., frequency of correctly written digits for single-digit multiplication problems), compound accuracy (i.e., percentage of correctly written digits for multi-digit multiplication problems), and compound fluency (i.e., frequency of correctly written digits for multi-digit multiplication problems).

Data Analysis

Descriptive statistics were used to summarize the multiplication performance of fifth graders in each individual country. Independent sample t-tests were then conducted to examine whether any difference existed in element and compound skills using different measurements (fluency and accuracy). Pearson correlation coefficients were also used to analyze relationships among the variables in each country and among all the three countries. Additionally, we compared the differences in student performance across countries via multiple t-tests.

Interscorer Agreement

An independent scorer counted correct and total digits every tenth test packet across three test sections. The mean interscorer agreement across the three sets was 97.9%. The mean for element skill was 99.3% (range 90%-100%), and compound skill was 94.9% (range 43%-100%).

Results and Discussion

Question 1: How do fifth grade students from three different countries (i.e., United States, Japan, and Taiwan) differ in their performance with single digit and multi-digit multiplications problems?

***Accuracy* -** Students across all three groups exhibited high levels of accuracy in single-digit multiplication tests (Table 1, range from .78 to 1.00, with mean of .9882 and standard deviation of .273). The t-test revealed differences among the three groups: while student performances in Taiwan and in United States were similar, the students' accuracy performance in

Japan was significantly better in single-digit multiplication (Table 4, $\rho < .05$). However, the majority of the fifth graders successfully responded to the question items with few errors. Across all three groups, the mode was 1.00 (Table 1), and more than 90% of the students achieved 90% correct or better, the independent level of practice (Table 2). The average of Japanese student accuracy was .9944 correct, while Taiwan was .9892 and United States was .9818. Regardless of its statistical significance, the difference between groups may not represent educationally significant values.

Table 1.
Mean, Range, and Standard Deviation Comparison

Variables		N	Mini	Maxi	Median	Mode	Mean	SD
Element Accuracy	Japan	96	.94	1.00	1.00	1.00	.9944	.0122
	Taiwan	95	.91	1.00	1.00	1.00	.9892	.0183
	United States	98	.78	1.00	1.00	1.00	.9818	.0406
	Overall	289	.78	1.00	1.00	1.00	.9882	.0273
Element Fluency (digits)	Japan	96	29	132	66.00	63.00	69.34	16.10
	Taiwan	95	21	128	72.00	52.00	68.63	23.83
	United States	98	16	112	52.50	41.00	45.50	19.12
	Overall	289	16	132	63.00	52.00	64.42	20.86
Compound Accuracy	Japan	96	.40	1.00	.94	1.00	.9077	.1118
	Taiwan	95	.58	1.00	.95	1.00	.9264	.0886
	United States	98	.23	1.00	1.00	1.00	.8937	.1644
	Overall	289	.23	1.00	.95	1.00	.9091	.1265
Compound Fluency (digits)	Japan	96	8	54	28.00	25.00	28.88	10.34
	Taiwan	95	4	90	33.00	35.00	34.21	17.46
	United States	98	1	42	15.50	6.00	17.04	10.16
	Overall	289	1	90	25.00	25.00	26.61	14.89

Although the overall average in multi-digit multiplication accuracy remained above .90 correct (Table 1, range from .23 to 1.00, with mean of .9091), the students' performance in multi-digit multiplication, was less accurate than single-digit multiplication and has much larger variances (Table 1, with standard deviation of .0237 in element accuracy and .1265 in compound accuracy).

Table 2.
Comparison with Performance Aims

Variables	Japan Frequency (Percent)	Taiwan Frequency (Percent)	United States Frequency (Percent)
Element Accuracy			
<90%	0 (0%)	0 (0%)	5 (5.10%)
=, >90%	96 (100%)	95 (100%)	93 (94.90%)
Element Fluency			
<80	72 (75%)	65 (68.42%)	84 (85.71%)
=, > 80	24 (25%)	30 (31.58%)	14 (14.29%)
Compound Accuracy			
<90%	32 (33.33%)	29 (30.53%)	33 (33.67%)
=, >90%	64 (66.67%)	66 (69.47%)	65 (66.33%)
Compound Fluency			
<40	78 (81.25%)	59 (62.11%)	97 (98.98%)
=, > 40	18 (18.75%)	36 (37.89%)	1 (1.02%)

Table 3 shows the significant differences between element and compound accuracy within each of the three groups ($\rho < .01$). Yet no significant differences were found between groups (Table 4). This finding suggests that many 5th graders across three groups performed as if they were still in the process of acquiring the skill despite being introduced to multi-digit multiplication a year before the data were collected. It was also notable that less than 70% of the participating students achieved the independent practice level of .90 correct (Table 2). All of the teachers in the participating schools had confirmed that multi-digit multiplication had been taught prior to the tests. The data support the proposition that dysfluent element behaviors could contribute to an increased amount of practice trials necessary for obtaining accuracy of the compound behavior (Johnson & Layng, 1992).

Table 3.
Paired Sample T-test Comparison between Skills

Variables	t	df	ρ.
All students (n=289)			
Element Accuracy - Compound Accuracy	11.366	288	.000
Element Fluency - Compound Fluency	42.879	288	.000
Japan (n=96)			
Element Accuracy - Compound Accuracy	7.584	95	.000
Element Fluency - Compound Fluency	28.750	95	.000
Taiwan (n=95)			
Element Accuracy - Compound Accuracy	7.153	94	.000
Element Fluency - Compound Fluency	19.499	94	.000
United States (n=98)			
Element Accuracy - Compound Accuracy	5.864	97	.000
Element Fluency - Compound Fluency	28.775	97	.000

Note: n=289

Fluency **-** The students in Japan and Taiwan performed significantly better than students in the United States with regard to multiplication element and compound fluency (Table 1 and Table 4). While no difference was found in element fluency between Taiwan and Japan, there was a notable difference between United States and Taiwan, and between United States and Japan (Table 4, $\rho < .01$). The differences in multi-digit multiplication were particularly large. On average, the students in Taiwan produced twice as many correct responses as those in United States while those in Japan correctly answered one and half times more multiplication digits than those in United States. The performance gaps apparently become larger with the increase of skill difficulty levels. The finding is consistent with the results from large scale international comparison (TIMSS, 2004) that Asian students are more likely to possess high frequencies with their basic computation skills and therefore better performance can be expected in their compound skills.

Table 4.

Independent Sample T-Test Comparison between Groups

Variables	t	df	ρ.
Element Accuracy			
Japan v. Taiwan	2.359	189	.019
US v. Taiwan	-1.784	191	.076
US v. Japan	-3.109	192	.002
Element Fluency			
Japan v. Taiwan	.242	189	.809
US v. Taiwan	-4.229	191	.000
US v. Japan	-5.451	192	.000
Compound Accuracy			
Japan v. Taiwan	-1.287	189	.200
US v. Taiwan	-1.715	191	.088
US v. Japan	-.688	192	.492
Compound Fluency			
Japan v. Taiwan	-2.572	189	.011
US v. Taiwan	-8.385	191	.000
US v. Japan	-8.047	192	.000

Question 2: Do measures of accuracy and fluency distinguish student performance?

Both accuracy and fluency measurements distinguished student performance. We could clearly distinguish student performance by the various levels of accuracy within and between the three groups of students in compound skill. However, the differences in element skill accuracy might not be educationally significant when the majority of students in all three groups achieved the level of independent practice (.90 correct). Although statistical tools could still be used to distinguish student performance, intensive practices to continuously strengthen performance accuracy are less likely to be available beyond the independent practice level. This finding illustrates one of the inherent problems relying on percentage as a metric for decision making. As Kubina and Morrison (2000) indicate, percentage can mask students' performance because it fails to provide any temporal information. Furthermore, students' performance cannot exceed 1.00 correct. For example, both students achieved 100% correct in their element accuracy; one produced 112 digits per minute while the other produced only 19 digits per minute. Accuracy measurement alone, in this example, was not useful in identifying those who were at risk of lack of computation automaticity. Due to this ceiling effect, accuracy measurements did not suggest meaningful differences in student performance within and across groups, which can be readily adapted in classroom practices. Beyond 100% accuracy, an alternative measurement shall be used to continue monitoring student performance.

By measuring fluency with frequency, students' performance was easily comparable in both skills. Notably, the frequency measures show vast differences of overall student performance between groups (Table 1). Research suggests adapting fluency aims/standards during fluency building. When compared with suggested fluency aims in single digit multiplication (Mercer, et al., 1982; Wood, Burke, Kunzelmann, & Koenig, 1978), 14.29% of the students in United States, 25% in Japan, and 31.58% in Taiwan achieved the lower end of fluency aim of at least 80 digits per minute (Table 2). On the other hand, only 1.02% of the students in United States, 18.75% in Japan, and 37.89% in Taiwan achieved the aim of 40 correct digits per minute in multi-digit multiplication (Lin, et al., 2005) (Table 2). The differences between groups were distinct and suggested educationally significant differences regarding students' current levels of competence.

In regard to the relationship between the two measurements, accuracy should also be viewed as the prerequisite for the development of fluency. Although high levels of accuracy do not directly contribute to high level of fluency (i.e., little or no relationship between accuracy and fluency was expected when the student had achieved high levels of accuracy), response speed can't be built up without the sufficiency accuracy. As shown on Table 1, 2 and 4, students in United States were less accurate in their element skill. It is not surprising that a significant positive correlation between element accuracy and fluency was only found within the group of students in United States (Table 6, $r = .520$). Students in United States still struggling with acquisition also have response frequencies further behind their peers from Taiwan and Japan. The data support previous findings that those who have not achieved high levels of accuracy tend to perform worse under time pressure (Rhymer, Skinner, Henington, D'Reaux, & Sims, 1998).

Question 3: How is the element skill performance (single digit multiplications) associated with compound skill performance (multi-digit multiplications)?

Considering the sequential nature of mathematics, students must be proficient with element skills in order to proceed successfully through the compound content (US Department of Education, 2008). Therefore, element skill performance was expected to positively influence student performance in compound skill. To be more specific, it was hypothesized that the better an individual performed in their element accuracy, the better one could expect in his/her compound accuracy. The same was expected with element and compound fluency.

Element fluency (single-digit multiplication) was the single most powerful predictor for compound fluency (multi-digit multiplication) (Table 5, $r = .696$), which supports previous findings (Lin et al., 2005). The more fluent the students were in their basic skills, the more likely they achieved competence with a directly related advanced skill such as multi-digit multiplication. The result was consistent in each of the three groups (Table 6). Although the prediction power may be weakened for those who had performed equal and/or above element fluency aims, the findings support the hypothesis of element skills requiring a certain frequency necessary for the successful creation of the compound skill (e.g., Binder, 1996; Bucklin, et al., 2000; Evans, et al., 1983; Evans, et al., 1985; Haughton, 1980; McDowell, et al. 2001, 2002).

Accuracy measurement alone, however, did not serve as a meaningful predictor. The statistical significance of correlation coefficient was only found in data collected in Taiwan and United States (Table 6) in which student performance in element accuracy was slightly less accurate than that in Japan (Table 1 and 4). The overall correlation coefficients in accuracy pairs (Table 5, r = .396) were less than fluency pairs. Because variances are expected diminished when the overall student performance improves, this finding further confirmed that students in Japan performed better in accuracy. Specifically in Japan, the smaller variances did not establish any powerful correlation. In addition, considering that student element accuracy in general was very high, it suggested that when students had acquired their basic skill, accuracy measurement was no longer visually sensitive enough to identify those who might still experience difficulties in their more advanced skills. Using frequency as a measure was more appropriate when the target response level was beyond acquisition.

Table 5.
Intercorrelations between Fluency and Accuracy

Variables	1	2	3	4
1. Element Accuracy	--	.261**	.396**	.202**
2. Element Fluency		--	.162**	.696**
3. Compound Accuracy			--	.321**
4. Compound Fluency				--

Note. n=289 , * ρ < .05 (two-tailed), ** ρ < .01 (two-tailed)

Table 6.
Intercorrelations between Fluency and Accuracy for Students in Japan, Taiwan, and United States

Variables	1	2	3	4
Japan (n=96)				
1. Element Accuracy	--	.135	.040	.157
2. Element Fluency		--	.038	.528**
3. Compound Accuracy			--	.409**
4. Compound Fluency				--
Taiwan (n=95)				
1. Element Accuracy	--	.087	.268**	.028
2. Element Fluency		--	.073	.693**
3. Compound Accuracy			--	.276**
4. Compound Fluency				--
United States (n=98)				
1. Element Accuracy	--	.359**	.520**	.303**
2. Element Fluency		--	.271**	.756**
3. Compound Accuracy			--	.385**
4. Compound Fluency				--

* ρ < .05 (two-tailed), ** ρ < .01 (two-tailed)

Conclusion

This research study examined the performance differences among students from the United States, Taiwan, and Japan. By examining the differences between two levels of multiplication, this study supported the concept of application and showed that fluency with element multiplication skills (i.e., single digit multiplication) explained the performance variances in a multiplication compound skill (i.e., multi-digit multiplication). The results confirm that students from Taiwan and Japan are more fluent than students in America, a finding similar to others showing Asian students with more advanced arithmetic and advanced math skills (Geary, et al. 1999; TIMSS 2004; US Department of Education, 2008). Additionally, the element skill of basic multiplication fact fluency was the most powerful predictor for compound fluency, again a finding that replicates previous research (Lin & Kubina, 2005). Accuracy alone, on the other hand, was not sufficient for distinguishing performance variances. Due to the ceiling effect, when the majority of the students achieved high level of accuracy, it is no longer sensitive to small changes. In addition, it does not provide a useful predictor for future performance in compound skills. This finding is meaningful because most teachers almost exclusively use percent correct, an accuracy score, for judging competence (Kubina & Morrison, 2000). Furthermore, application is a testable model where frequency, similar to activation energy in a chemical process, allows teachers to more precisely diagnose student problems in basic and advanced multiplication.

Future Research Questions

This study, correlational in nature, brings forth some valuable possibilities for future research questions. Most notably, what accounts for the differences in both element (single digit) and compound (multi-digit) multiplication fluency? What procedures account for the higher fluency scores? Furthermore, do other math skill fluencies predict similar outcomes? For the concept of application to be useful research should confirm the its powerful predictive ability as it has in this and previous studies (e.g., Lin & Kubina, 2005).

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