Behavior Analysis in Practice (2021) 14:608–622
https://doi.org/10.1007/s40617-020-00501-3

Building Prealgebra Fluency Through a Self-Managed Practice Intervention: Order of Operations

James D. Stocker Jr[1] · Richard M. Kubina Jr[2]

Accepted: 11 September 2020 / Published online: 16 February 2021

Abstract

Behavioral fluency refers to a combination of accuracy and speed that enables students to function proficiently in the learning environment. The present study investigated the effects of a self-managed frequency-building intervention on the behavioral fluency of a critical prealgebra skill in four 6th-grade students. The intervention involved students having access to the PEMDAS (parentheses, exponents, multiplication, division, addition, and subtraction) mnemonic during frequency building. Using an alternating-treatments design, the first experimental condition presented the intervention as three 1-min practice trials with 30 s of feedback delivered immediately after each frequency-building trial ended. The second condition offered one 3-min practice trial with 90 s of feedback once the trial ended. A baseline condition (no practice) had the students engage in a 1-min timed trial with no feedback. The alternating-treatments design demonstrated that three of the four students produced a superior performance within the two intervention conditions when compared to baseline. However, the results did not conclusively show that one frequency-building intervention was superior to the other.

Keywords frequency building · mathematics fluency · pre-algebra fluency · complex computation · behavioral fluency · feedback · self-managed interventions

Employment statistics suggest education and training in science, technology, engineering, and mathematics (STEM) provide a path to careers with greater job security and higher wages in a rapidly growing sector of the global economy beset by labor shortages, especially in certain occupations and industry sectors of the skilled technical workforce (National Academies of Sciences, Engineering, and Medicine, 2017). Research indicates that mathematics achievement increases the probability of college matriculation and operates as the gatekeeper to STEM careers (Adelman, 2006; Wang 2013). However, a persistent failure to achieve proficient mathematical performance is rooted in a lack of component skill acquisition and mastery in a hierarchical progression. Beginning with element skill sets first introduced in prekindergarten, American students' mathematical repertoires are compounded by dysfluent composite skills as students advance into upper grade levels, resulting in the United States' ongoing difficulty supplying the STEM workforce (National Mathematics Advisory Panel [NMAP], 2008).

NMAP (2008) and the National Council of Teachers of Mathematics (NCTM, 2000) recommend educators in the elementary years focus on conceptual understanding and computational and procedural fluency with whole numbers. Development of these foundational skills is necessary to prepare students for later skill development

Research Highlights
• A behavioral fluency intervention for complex computation has the capacity to prepare middle school students for later success in high school algebra.
• Middle school students can plausibly self-manage a frequency-building intervention for complex computation with minimal teacher mediation.
• Typically used in the research literature to increase fluency with simple computation, frequency building with complex computation shows promise at the middle school level.
• For complex computations, frequency building can quickly and predictably expose error patterns that occur, and inform the design of separate fluency practice activities for element skills.

✉ James D. Stocker, Jr
stockerj@uncw.edu

1 Department of Early Childhood, Elementary, Middle, Literacy and Special Education, University of North Carolina, Wilmington, 601 S. College Road, Wilmington, NC 28403, USA

2 Department of Educational Psychology, Counseling, and Special Education, The Pennsylvania State University, 209 CEDAR Building, University Park, PA 16802, USA

with, all of which are prerequisites to successful participation in algebra. NMAP (2008) asserted that curriculum must simultaneously develop "conceptual understanding of mathematical operations, fluent execution of procedures, and fast access to number combinations that jointly support effective and efficient problem solving." (p. xix) Deficits in early element skills, however, can later affect the acquisition and mastery of subsequent complex skills (Kubina & Yurich, 2012; McTiernan et al., 2016) and by middle school, student performance in mathematics begins to drop significantly (McFarland et al., 2018).

To remedy issues confronting educators delivering the mathematics curriculum, the Common Core State Standards Initiative (CCSSI, 2018) produced standards that (a) support greater focus on fewer topics to deepen conceptual understanding, (b) link concepts across grade levels, (c) reinforce computational and procedural fluency, and (d) apply mathematical knowledge to problem solving. Expectations for complex computation and related problem solving continue to increase as students engage in middle school mathematics. Standards connected to fluency start at Grade 1 and continue through Grade 7. By the end of middle school, CCSSI (2010, p. 53) recommended students have the skills to

- work with radicals and integer exponents (8.EE);
- understand the connections between proportional relationships, lines, and linear equations (8.EE);
- analyze and solve linear equations and pairs of simultaneous linear equations (8.EE);
- define, evaluate, and compare functions (8.F); and
- use functions to model relationships between quantities (8.F).

Despite the importance of fluency in the mathematics curriculum, students often do not receive sufficient practice and move prematurely to the next-level skill before performing the prerequisite skill(s) fluently (Binder, 1996, 2003). A number of researchers have also suggested that the quality and quantity of curricular materials, as well as the instructional knowledge to effectively implement practice activities, do not support fluency instruction (Daly et al., 2007; NMAP, 2008; Witzel & Riccomini, 2007). As a result, conceptual understanding and fluency often do not develop synergistically and can have deleterious effects on mathematics achievement (Rittle-Johnson & Siegler, 1998; Rittle-Johnson et al., 2001).

Research on Mathematics Fluency and Complex Computation

For a new skill to be acquired, instructional activities focus on the quality and accuracy of the response (Archer & Hughes, 2011; Ardoin & Daly, 2007). Students should then engage in systematic practice that combines accurate respondingand appropriate speed to build fluency. A fluent performance occurs when a student can respond accurately to a specified number of problems at a specified level of difficulty within a set period of time (NCTM, 2000). Unfortunately, a paucity of research exists on interventions that increase fluency with complex computation. Simple computation represents a majority of the work related to fluency intervention research (Foegen et al., 2008; Geary et al., 2007).

In a meta-analysis of simple-computation mathematics-fluency interventions, Codding et al. (2011) reported a large effect size when the intervention consisted of practice with modeling or "drill," and when the intervention involved three or more components. Practice with modeling commonly refers to a student having the opportunity to review and receive feedback on the problem and answer. Immediate and corrective feedback has been shown to reinforce correct responding versus errors (Burns et al., 2008; Daly et al., 2007; Fuchs et al., 2008; Rivera & Bryant, 1992). Codding et al. also discovered that self-managed interventions produced a moderate effect size. Self-managed interventions show improvement in stimulus control, incentive, and independence (Hughes et al., 1991; Mace et al., 2001; McDougall & Brady, 1998; Reid et al., 2005).

Explicit, consecutive timings often function as an effective practice method to build fluency. For the most part, researchers and teachers quantify performance from timings via digits correct per minute (DCPM) or correct problems per minute to produce a frequency or rate of performance. Frequency denotes a count over a specified time of observation, which yields a more precise representation of student performance versus using a percentage correct (Johnson & Street, 2013; Merbitz et al., 2015). In the precision-teaching literature, *frequency building* refers to timed repetition of pinpointed behavior followed by immediate feedback (Kubina, 2019). A number of studies that have incorporated frequency building as the primary practice component reported an increase in speed, sustained accuracy, and endurance or resistance to fatigue (Beverly et al., 2009, 2016; Brady & Kubina, 2010; Bullara et al., 1993; Chiesa & Robertson, 2000; Datchuk, 2016; Fitzgerald & Garcia, 2006; MacDonald et al., 2006; McTiernan et al., 2016; Miller et al., 1995; Stocker et al., 2018; Strømgren et al., 2014).

To scaffold interventions, prior researchers have used mnemonic strategies and checklists as tools for modeling, reminding, and giving feedback (Maccini et al., 2007; Manalo et al., 2000; Mastropieri & Scruggs, 1989; Nelson et al., 2013; Stocker et al., 2018). Mnemonic strategies refer to words, rhymes, or sentences that aid students in the acquisition and recall of facts and procedures (Mastropierri & Scruggs, 1998). For complex computation in mathematics, students have traditionally used mnemonics to aid in solving for long division, two-binomial distribution, order of operations, and the metric system. Mnemonic strategies used alone lead to acquisition and accuracy when used with complex computation but may hinder procedural fluency (Johnson & Street, 2013).

In a recent study conducted by Stocker et al. (2018), four middle school students used mnemonics, checklists, and answer keys to self-manage a frequency-building intervention solving for long division, adding and subtracting fractions, and order of operations. Student participants had a skill assigned to either three 1-min timings with self-managed feedback after each timing, one 3-min timing followed by self-managed feedback, and a 1-min baseline without intervention. Results suggested students generally performed best when engaged in the most predictable algorithm (long division, followed by fractions, and then order of operations) despite the timing assigned to the algorithm. The anticipated treatment diffusion (e.g., practice effects) that occurred due to increased speed with simple computation and associated procedures highlighted the importance of accuracy and feedback with element skills associated with the algorithm. Element skills that presented the most difficulty involved (a) computing with decimals and positive and negative numbers, (b) changing improper fractions to mixed numbers, and (c) calculating remainders. Evidence gathered from 3 weeks of intervention and the retention measure indicated that students can self-manage feedback and self-correct during frequency building as indicated by the increase in the number of correct problems and the decrease in the number of incorrect problems over the span of the investigation.

Present Study

Middle school students apply element skills learned in elementary school to solve more complex computations. Complex problem solving in pre-algebra often involves different combinations of element skills (e.g., fractions, decimals, integers). Students who fluently execute pre-algebraic algorithms secure an advantage later in the high school algebra curriculum over students who have not reached a level of fluent performance (NMAP, 2008). Order of operations serves as a prime example of a complex pre-algebra skill that requires fluent execution. To examine the effects of frequency building on order of operations, the researchers posed the following questions:

1. What effect does a self-managed frequency-building intervention have on solving problems involving order of operations?
2. What performance differences occur when given three 1-min assessments versus one 3-min assessment?

Method

Participants and Setting

The selection of students was based on teacher nomination and parent response. Two female students (Stephanie and Grace) and two male students (Bob and Luno) from a sixth-grade class participated in the study. Located in a suburban Pennsylvania charter school, the intervention took place in a separate classroom where teachers conduct small-group instruction at different points during the day. None of the four students received special education services or exhibited significant mathematics deficits. The students scored between 32 DCPM (instructional level) and 41 DCPM (mastery level) on a curriculum-based assessment (Deno & Mirkin, 1977). The students also received prior instruction during regularly scheduled mathematics instruction using the PEMDAS (parentheses, exponents, multiplication, division, addition, and subtraction) mnemonic to solve order of operations problems. At the time of the study, the school did not have an adopted program or curriculum to support frequency building.

Materials

Student materials comprised experimenter-designed (a) mnemonic PEMDAS cue cards, (b) practice sheets, (c) the corresponding answer keys for feedback, and (d) assessments. Ancillary materials included (a) pencils and erasers, (b) small rewards for participating, (c) procedural integrity checklists, (d) an assessment schedule, (e) instructions, (f) a stopwatch, and (g) an application called PrecisionX (CentralReach, 2019) for recording, displaying, and analyzing data.

Three exclusive sets of practice sheets with corresponding answer keys and assessments concentrated on either fractions, decimals, or integers. Individual practice worksheets and assessments contained nine order of operations problems. Each problem contained number expressions (e.g., 4×3) or mixed-operator expressions (e.g., $2 + 5 - 5$). The mixed-operator expressions did not include multidigit leading to multistep complex computation (e.g., 43x14 + 20; $6 \cdot 54 + 84$). To balance the level of difficulty between symbols and digits on practice worksheets and assessments, the study adopted the following decision rules to represent PEMDAS:

- 18 sets of parentheses total, 2 per problem
- 9 exponents total, 1 per problem, with products of 81 or less (i.e., 1 to 9 squared)
- 5–8 multiplication (x) math facts per assessment, no more than 2 per problem
- 5–8 division (÷) math facts per assessment, no more than 2 per problem
- 5–8 addition (+) math facts per assessment, no more than 2 per problem

- 5–8 subtraction (-) math facts per assessment, no more than 2 per problem

Response Measurement and Accuracy

Dependent Variables

The dependent variables included the number of correct written digits and symbols per minute (CDSM) and the number of incorrect written digits and symbols per minute (IDSM) a student made during a 1-min timed interval. A correct digit snd symbol signified an accurate written presentation of numbers and symbols. For instance, (12) + (1/2) yields 10 correct digits and symbols. An incorrect digit and symbol denoted an (a) illegibly written digit, (b) an inaccurate numerical presentation, or (c) a digit disconnected from the process of reaching the solution. Individual students completed three 1-min interval assessments per day over a period of 10 days—one for baseline, one after Intervention 1, and one after Intervention 2 (See Table 2). Each assessment contained nine problems with more potential digits than a student could complete in the allotted time.

Accuracy

Accuracy denotes the degree to which observed values provide a true representation of the events that transpired in an experiment (Kostewicz et al., 2016). Accuracy delivers more precise information than interobserver agreement in relation to the accuracy and reliability of experimental data (Johnston & Pennypacker, 2009). In the present investigation, the experimenter and graduate student independently corrected and subsequently cross-referenced the results of the assessments in order to accurately represent the true value of the dependent variable, rather than employing interobserver agreement where an associate corrects a fraction of the assessments. Hence, the dependent variable in the current study reflects 100% accuracy.

Independent Variable and Procedural Integrity

Independent Variable

The present study included a baseline condition (i.e., no practice or frequency building) and two independent variables (i.e., Intervention 1 and Intervention 2) using the same self-managed order of operations intervention. Baseline (no intervention) facilitated the comparison and evaluation of the other two interventions. Intervention 1 involved three 1-min frequency-building trials repeating the same practice worksheet. After each 1-min timing, the students checked their work using an answer key for 30 s. Intervention 2 involved one 3-min frequency-building trial. Afterward, the students checked their work using an answer key for 90 s. The students had visual access to the PEMDAS mnemonic cue card during the two intervention conditions.

Procedural Integrity

The experimenter used a procedural integrity checklist to document the precise and consistent implementation of the interventions. The checklist ensured the readiness and appropriate placement of practice worksheets, assessments, answer keys, and PEMDAS cue cards; the proper sequence for reading the instructions and distributing the intervention materials; and the accuracy of timings. On 3 separate days, a graduate student conducted a check for procedural integrity. A 2-hr training session for the graduate student occurred before the first day of the intervention. The training comprised reviewing the materials, as well as a simulation of the intervention and the procedural integrity check process. Calculating procedural integrity consisted of dividing the number of steps correctly completed by the total number of possible steps and multiplying by 100 (Ledford & Gast, 2009). The mean procedural integrity came to 100%.

Experimental Design

The current study employed an alternating-treatments design (Cooper et al., 2020; Johnson & Pennypacker, 2009; Kazdin, 2011) to examine the effects of the two intervention conditions. As suggested by the name of the design, the two frequency-building interventions and baseline alternated systematically in order to isolate the influence of the independent variable assigned to the different conditions (Kazdin, 2011). To alternate interventions in the current study, the experimenter (a) randomly assigned three discrete skills to each student (see Table 1) and (b) counterbalanced the order in which the students received the three conditions (see Table 2). When the three separate skills are randomly assigned, the design eliminates confounds that could occur when students share the same skill in each condition. By counterbalancing the three conditions, the experimenter attempted to control for the effects of a static order that may favor one condition over the other. The current study recruited four participants for three conditions. As a result, Grace and Bob shared fractions for baseline, Stephanie and Grace shared

Table 1 Intervention Assignments

Student	Baseline	Intervention 1	Intervention 2
Stephanie	Integers	Fractions	Decimals
Grace	Fractions	Integers	Decimals
Bob	Fractions	Decimals	Integers
Luno	Decimals	Fractions	Integers

Note. Baseline = no practice; Intervention 1 = three 1-min trials; Intervention 2 = one 3-min practice trial.

Table 2 Intervention Schedule

Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7	Day 8	Day 9	Day 10
BL, 1, 2	1, 2, BL	2, BL, 1	BL, 1, 2	1, 2, BL	2, BL, 1	BL, 1, 2	1, 2, BL	2, BL, 1	BL, 1, 2

Note. BL = baseline; 1 = Intervention 1; 2 = Intervention 2.

decimals for Intervention 1, and Bob and Luno shared integers for Intervention 2.

Procedure

Prior to students' arrival at the experimental setting, the experimenter arranged the first set of materials (e.g., baseline assessments or practice sheets with answer keys and mnemonic cue cards) face down on a large rectangular conference table. When the students entered the room, they sat in specified seats and listened to the first of a series of instructions corresponding to the day's alternating intervention schedule. The instructions requested the students to (a) start with the first problem and work across the page, (b) not skip problems, (c) show all work, and (d) work as quickly as possible. In a scenario where a student stopped working before the timer expired or had a question, the experimenter would state to the student "Keep doing the best work you can."

On the first day, the students started with the 1-min assessment for baseline. The students then handed in the baseline assessment and immediately received the packet for Intervention 1. The students attended to instructions for Intervention 1 and worked on the first of three 1-min practice sheets. During frequency building, the students could view their mnemonic cue cards. The students kept the mnemonic cue card off the paper to not obstruct the view of the problem. Following the first frequency-building trial, the students evaluated their work with an answer key for 30 s. The students then handed in the first practice sheet, turned over the answer key, and repeated the same process an additional two times. The three 1-min trials produced a total of 90 s of self-feedback. Next, the students completed a 1-min assessment for Intervention 1 without the PEMDAS mnemonic cue card.

For Intervention 2, the students worked on a practice sheet for 3 min with access to the mnemonic cue card. Afterward, they checked their work for 90 s with an answer key and then completed the final 1-min assessment for Intervention 2. The experimenter then made general, positive comments about their overall efforts and thanked the students for their participation. After the students left the room, the experimenter wrote anecdotal notes based on observations. Example notes included absences, questions students asked, and observable behavior (e.g., student not showing work, student unfocused). The experimenter then promptly collected, scored, and input the data. Each student received 10 days of the three conditions.

Data Display

The experimenter recorded, evaluated, and visually displayed data on segments of standard celeration charts (SCCs; Graf & Lindsley 2002; Lindsley, 2005). Figures 1 and 2 represent key elements of the SCC. The critical features of the SCC include changes in behavior (a) recorded in calendar time, (b) displayed proportionally, and (c) quantified to yield precise, quantitative measures. The following measures augment visual analysis on the SCC: level, celeration, and improvement index.

Level

Level represents the mean performance for both CDSM and IDSM. One method for determining the level involves using the geometric mean (Kubina, 2019). The geometric mean provides a measure that normalizes the range of numbers calculated; the geometric mean does not weigh or prefer one set of numbers over another set of numbers. The geometric mean also mitigates the effects of outliers that can skew data (Clark-Carter, 2005). A 60% change in performance from 2 to 5 CDSM reflects the same proportional effect of a 60% change in CDSM from 20 to 50.

Level Multiplier

In the present study, the researchers employed a level multiplier (Kubina, 2019) to quantify the difference in levels (mean performance) of CDSM to CDSM and IDSM to IDSM between baseline and the experimental conditions. The calculation includes dividing the larger value by the smaller value. The quotient then takes on the multiplication or division sign of the greater initial value depending on the positions of the two compared levels. For example, a student produces a level of 8 IDSM during baseline and a level of 3 IDSM during an experimental condition. The level multiplier or change in average performance between the student's baseline and experimental condition performance equals a ÷2.7 (62.5%) difference in IDSM (i.e., 8 ÷ 3 = 2.7; the division sign is applied because from baseline to intervention the errors show a reduction).

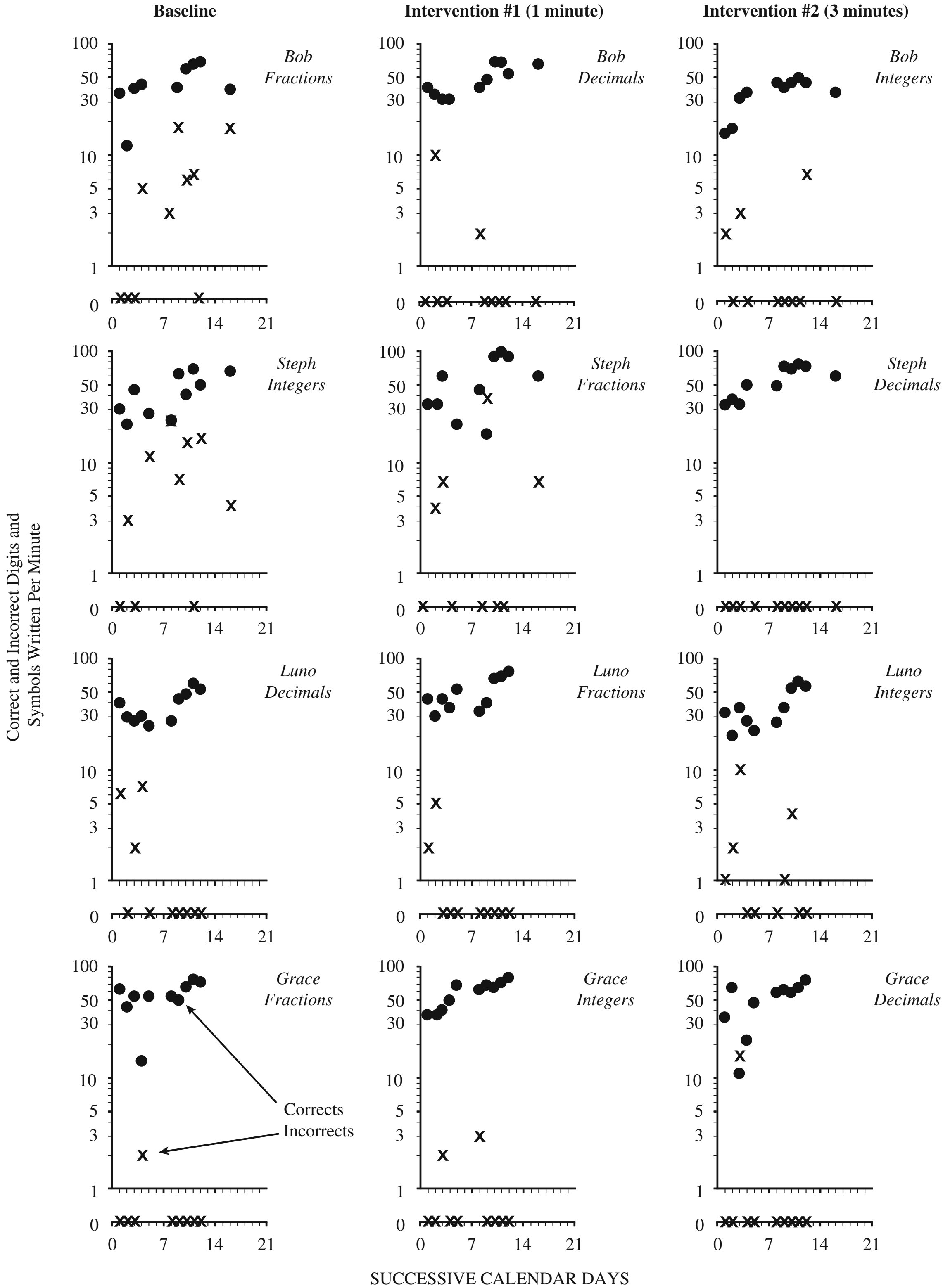

Fig. 1 Correct Digits/Symbols and Incorrect Digits/Symbols

Celeration

Celeration refers to a standard unit of measurement that quantifies a change in frequency or rate of performance over time (Johnston & Pennypacker, 2009). For instance, a student who solves 40 CDSM on Monday's assessment and then accelerates to 60 IDSM on the following Monday's assessment will produce a celeration value of

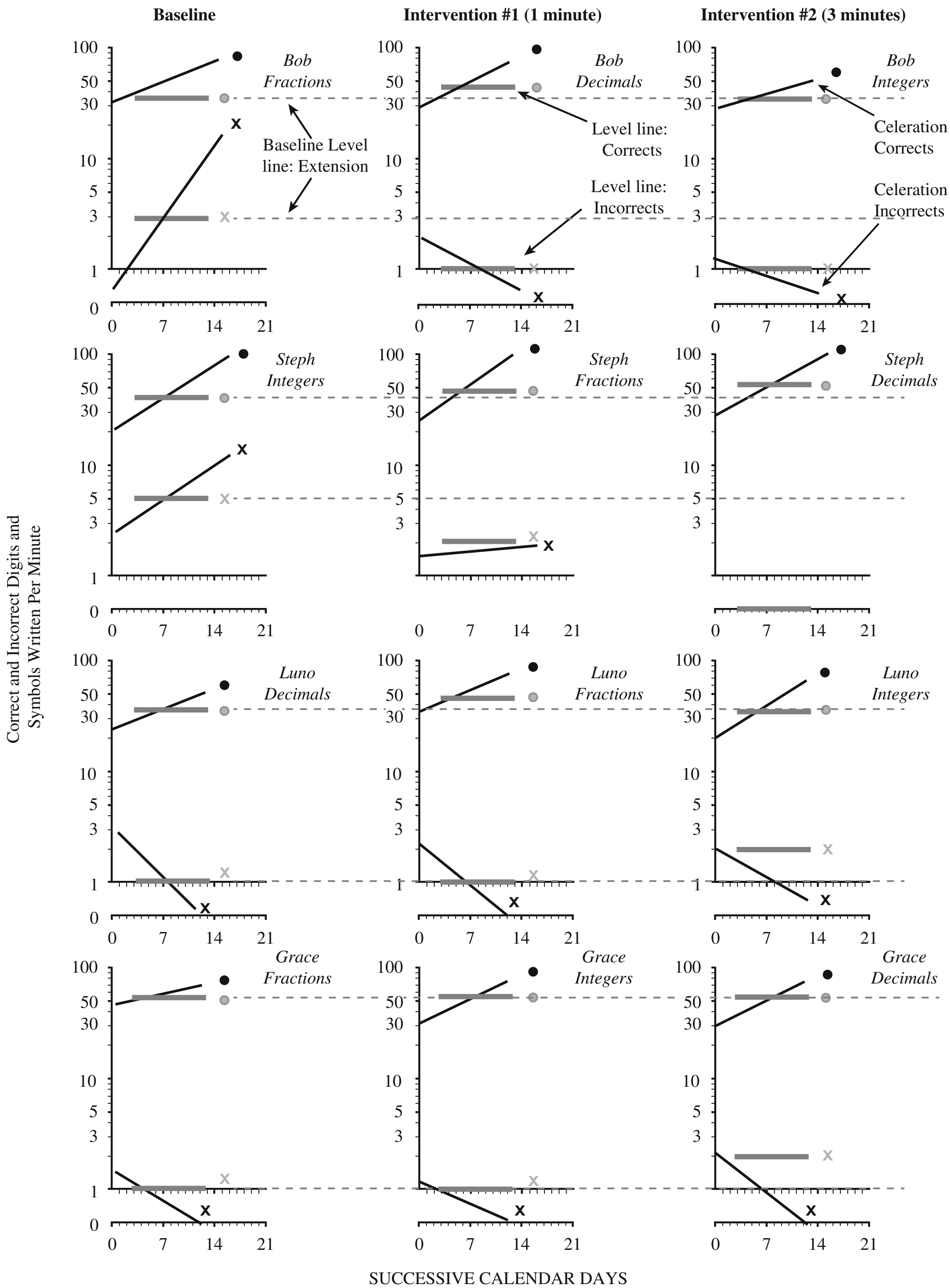

Fig. 2 Level Lines and Celeration Lines

×1.5 per week—a 50% weekly growth rate. Another student in the same class who accelerates from 40 CDSM to 80 CDSM will double their performance, thus producing a celeration value of ×2.0 per week, or a 100% weekly growth rate. Similarly, the SCC also assigns celeration values quantifying a decrease in the frequency of performance. A student who produces 15 IDSM on Monday's assessment and decelerates to 10 IDSM on the following Monday's assessment will produce a celeration value of ÷1.5 per week, or a 33% decay rate for errors.

Celeration Multiplier

Like the level multiplier, the celeration multiplier also quantifies differences between CDSM to CDSM and IDSM to IDSM with baseline and the experimental conditions. However, the celeration multiplier differentiates speed of change (Kubina, 2019). Take, for example, a student with a celeration of ×1.1 per week for CDSM in baseline who has a celeration of ×2.2 per week during the experimental condition. The celeration multiplier calculation must account for not only speed differences but also the directions of the celerations between conditions. Therefore, the following rules apply: if both values have the same sign (i.e., both multiplication or both division), divide the larger value by the smaller value and apply the sign signifying the comparison of the change (i.e., if the resulting change from baseline to intervention sped up, a multiplication sign would appear; for instances where the speed decayed, the value has a division sign). However, for celerations with different signs (multiplication to division, or division to multiplication) the rule states to multiply the values together and use the sign representing the speed difference (multiplication for an accelerating speed difference and division for a decelerating speed difference). For example, a speed comparison of ×1.1 per week in baseline and ×2.2 per week in an intervention condition yields 2.2 ÷ 1.1 = 2, with a multiplication sign for ×2, stating the speed comparison of the intervention value occurred twice as fast as the baseline condition.

Improvement Index

The improvement index provides a metric for the degree of progress improvement (Kubina, 2019). To find the improvement index, the individual obtains the ratio of concurrent celerations; the higher the improvement index value, the more progress improvement has occurred. For instance, an improvement index of ×2.0 indicates progress has advanced by 100%, or has doubled over the span of the experimental condition. Conversely, an improvement index of ÷2.0 indicates that progress has worsened by half (50% reduction) over the span of the experimental condition. For educators, the improvement index provides a practical, numerical summation that reflects the magnitude of progress made during an intervention or prescribed practice activity (Kubina & Yurich, 2012).

Improvement Index Change

The improvement index change compares two index measures between conditions (Kubina, 2019). In the present experiment, the conditions examined were baseline, Intervention 1,and Intervention 2. The resulting ratio of ratios offers a value quantifying progress change from baseline to intervention. The formulas for the improvement index change follow the same logic as the celeration multiplier. If both values have the same sign (i.e., both multiplication or both division), divide the larger value by the smaller value and apply the sign signifying the comparison of the change (i.e., improved progress from baseline to intervention necessitates a multiplication sign, whereas worsening progress would have a division sign). When improvement index values have different signs, the rule says to multiply the values together and use the sign representing the progress difference: again, as previously stated, a multiplication sign for improved progress and a division sign for worsened progress.

Results

Table 3 includes all of the individual performance outcomes for level corrects, celeration corrects, level incorrects, celeration incorrects, and improvement index values. Table 4 provides a comparison analysis for baseline versus Intervention 1 and baseline versus Intervention 2. Figure 1 displays the trend on SCC segments. The dots on the chart segments denote correct performance frequencies (e.g., number of CDSM). The X's represent incorrect performance frequencies (e.g., number of IDSM).

Figure 2 shows the level lines and celeration lines as they would appear on the data in Figure 1. The black lines (celeration lines) provide a visual representation of the growth or decay of performance frequencies. The lines followed by the dot depict CDSM, whereas the X's refer to IDSM. The gray lines indicate the level, determined by the geometric mean, of CDSM and IDSM for each condition and student: again, the dots and X's standard for CDSM and IDSM. Celeration and level lines presented together provide a visual reference for the trend and average performance for the separate conditions in the alternating-treatments design.

Student Performance Outcomes

Level of CDSM

Bob maintained similar levels between skills, with 37 CDSM for fractions (baseline), 39 CDSM for decimals (Intervention 1), and 35 CDSM for integers (Intervention 2). Steph had a level correct of 41 CDSM for integers (baseline), 48 CDSM for fractions (Intervention 1), and 53 CDSM for decimals (Intervention 2). Grace had similar levels of 53 CDSM for fractions (baseline) and 54 CDSM for integers (Intervention 1). Grace produced 44 CDSM for decimals (Intervention 2). Luno produced levels of 38 CDSM for decimals (baseline), 47 CDSM for fractions (Intervention 1), and 35 CDSM for integers (Intervention 2).

Table 3 Student Performance Indicators

	Condition	Skill	CDSM Level	CDSM Celeration	IDSM Level	IDSM Celeration	II
Bob	Baseline	Fractions	37	×1.4	3	×3.9	÷2.7
	Intervention 1	Decimals	46	×1.4	1	÷1.7	×2.4
	Intervention 2	Integers	35	×1.5	1	÷1.2	×1.8
Steph	Baseline	Integers	41	×1.6	5	×2.4	÷1.5
	Intervention 1	Fractions	48	×1.5	2	×1.1	×1.4
	Intervention 2	Decimals	53	×1.5	0	×1.0	×1.5
Luno	Baseline	Decimals	38	×1.5	1	÷3.0	×4.5
	Intervention 1	Fractions	47	×1.5	1	÷2.4	×3.6
	Intervention 2	Integers	35	×1.7	2	÷1.8	×3.0
Grace	Baseline	Fractions	53	×1.3	1	÷1.2	×1.6
	Intervention 1	Integers	54	×1.6	1	÷1.1	×1.8
	Intervention 2	Decimals	44	×1.9	1	÷1.8	×3.4

Note. CDSM = correct digits and symbols per minute; IDSM = incorrect digits and symbols per minute; II = improvement index.

Celeration of CDSM

Celerations for CDSM ranged from ×1.3 per week (30% weekly change rate) to ×1.9 per week (90% weekly change rate) over all conditions of the experiment. Bob, Grace, and Luno produced the largest gain in celeration for CDSM via Intervention 2 (3-min practice trials). Steph accelerated at ×1.6 per week (60% weekly growth) in the baseline condition and performed similarly at ×1.5 per week (50% weekly growth) during Interventions 1 and 2. Grace exhibited significant gains in performance across all three conditions, posting a consistent acceleration of ×1.3 per week (30% weekly growth in baseline), ×1.6 per week (60% weekly growth in Intervention 1), and ×1.9 per week (90% weekly growth in Intervention 2). Luno produced equal celerations during the baseline and Intervention 1 conditions, at ×1.5 per week (50% weekly growth), and produced a celeration of ×1.7 per week (70% weekly growth) in Intervention 2. Bob also showed identical celeration for the baseline and Intervention 1 conditions, with a celeration of ×1.4 per week (40% weekly growth), and produced an increase of ×1.5 per week (50% weekly growth) for Intervention 2.

Table 4 Level, Celeration, and Improvement Index Analysis

		Skill	Level Multiplier CDSM	Celeration Multiplier CDSM	Level Multiplier IDSM	Celeration Multiplier IDSM	Improvement Index Change
Bob	Baseline	Fractions	*37*	*×1.5*	*3*	*×3.9*	*÷2.7*
	Intervention 1	Decimals	×1.2	×1.0	÷3.4	÷6.6	×7.8
	Intervention 2	Integers	÷1.1	×1.1	÷3.4	÷4.7	×4.9
Steph	Baseline	Integers	*41*	*×1.6*	*5*	*×2.2*	*÷1.4*
	Intervention 1	Fractions	×1.2	÷1.1	÷2.5	÷2.0	×2.1
	Intervention 2	Decimals	×1.3	÷1.1	÷5.0	÷2.2	×2.1
Luno	Baseline	Decimals	*38*	*×1.5*	*1*	*÷3.0*	*×4.5*
	Intervention 1	Fractions	×1.2	×1.0	÷1.0	×1.3	÷1.3
	Intervention 2	Integers	÷1.1	×1.1	×2.0	×1.7	÷1.5
Grace	Baseline	Fractions	*53*	*×1.3*	*1*	*÷1.2*	*×1.6*
	Intervention 1	Integers	×1.0	×1.2	÷1.0	×1.1	×1.1
	Intervention 2	Decimals	÷1.2	×1.5	÷1.0	÷1.5	×2.1

Note. Italic text indicates original baseline celeration; CDSM = correct digits and symbols per minute; IDSM = incorrect digits and symbols per minute.

Level of IDSM

Steph and Bob produced the largest IDSM levels of five IDSM (integers) and three IDSM (fractions) during the baseline condition, respectively. Steph had a level of two IDSM (fractions) for Intervention 1 and a level of zero IDSM (decimals) for Intervention 2. Bob's IDSM level during both Interventions 1 and 2 (decimals and integers) came to one. Luno produced a level of one IDSM during baseline (decimals) and Intervention 1 (fractions), and two IDSM (integers) during Intervention 2. Grace had a level of one IDSM per condition.

Celeration of IDSM

Steph and Bob demonstrated an increase in IDSM during the baseline condition: ×2.4 per week (140% weekly growth) and ×3.9 per week (290% weekly growth), respectively. Visual analysis of the SCC segments (Figure 1) indicate that Steph's and Bob's performance during baseline exhibited the highest variability of all participants over the three conditions. Luno's baseline IDSM decelerated by ÷3.0 per week (67% weekly decay) to zero IDSM by Day 5. Grace emitted only two IDSM in the baseline condition on Day 4, producing a deceleration of ÷1.2 per week (17% weekly decay). During Intervention 1, Steph accelerated IDSM insignificantly by ×1.1 (10% weekly growth). Luno decelerated ÷2.4 per week (58% weekly decay) to zero IDSM by Day 3, whereas Grace decelerated ÷1.1 per week (9% weekly decay) after emitting two IDSM on Day 3 and three IDSM on Day 6. For Intervention 2, Steph exhibited 100% accuracy. Grace produced zero IDSM, with the exception of Day 3 (17 IDSM), generating a ÷1.8 per week (45% weekly decay), a decrease in IDSM. Bob and Luno decelerated IDSM by ÷1.2 per week (17% weekly decay) and ÷1.8 per week (45% weekly decay), respectively.

Improvement Index

The improvement index provides a metric of progress change (i.e., improving, worsening, or maintaining). In the baseline condition, Steph's and Bob's progress deteriorated by ÷1.5 (33% worsening progress) and ÷2.7 (63%), respectively. Conversely, Grace improved by ×1.6 (60% improving progress) and Luno by ×4.5 (350%). For Intervention 1, Steph and Grace generated an improvement of ×1.5 (50%). Bob and Luno produced an improvement of ×2.9 (190%) and ×3.6 (260%), respectively. For Intervention 2, Luno produced an improvement of ×3.6 (260%), whereas Grace generated a ×3.4 (240%) improvement. Bob's and Steph's improvement index demonstrated a ×1.5 (50%) progress improvement.

Comparison Analysis

Level Multiplier for CDSM

Steph had a higher level of CDSM, ×1.2 (20% rise in Intervention 1) and ×1.3 (30% rise in Intervention 2), when compared to baseline. Bob did not have significant differences in levels when compared to baseline. Grace also did not have a significant difference in level from baseline to Intervention 1, but her level from baseline to Intervention 2 indicated a ÷1.2 (17% drop) difference. Luno's average response rate as shown from the baseline to the Intervention 1 level differed by ×1.2 (20% higher for Intervention 1) but did not differ significantly (÷1.1) between baseline and Intervention 2.

Celeration Multiplier for CDSM

Bob, Steph, and Luno did not generate a significant increase or decrease in celeration from baseline to the intervention conditions. Bob and Luno shared a similar performance of ×1.0 from baseline to Intervention 1 and a ×1.1 (7% speed change) insignificant increase in celeration (i.e., speed of change) from baseline to Intervention 2. Steph decreased celeration insignificantly from baseline to both intervention conditions by ÷1.1. Grace showed a consistent increase from baseline to Intervention 1 and Intervention 2, posting ×1.2 (20% speed change) and ×1.5 (50% speed change) gains in CDSM.

Level Multiplier for IDSM

Bob and Steph exhibited robust decreases in IDSM for both intervention conditions when compared to baseline. Steph's IDSM dropped by ÷2.5 (60%) and ÷5 (100%) when comparing Intervention 1 and Intervention 2 to baseline, respectively. Bob's difference came to ÷3.4 (71%) for both experimental conditions when compared to baseline. Luno produced a similar level of IDSM when compared to baseline for Intervention 1 and showed a ×2 greater average rate of responding from baseline compared to Intervention 2. Grace remained at similar levels between the three conditions.

Celeration Multiplier for IDSM

Steph and Bob generated a robust decay in IDSM from baseline to the intervention conditions. Bob exhibited the largest decrease in IDSM of ÷6.6 (85% speed change) and ÷4.7 (79% speed change) when comparing his performance from Intervention 1 and Intervention 2 to baseline, respectively. Steph posted a significant decrease in IDSM of ÷2.2 (55%) during Intervention 1 and ÷2.4 (58%) during Intervention 2 when compared to baseline. Grace exhibited a similar decrease in IDSM between baseline and Intervention 1 conditions and a decrease of ÷1.5 (33%) between baseline and Intervention 2 conditions. Luno produced a significant increase in IDSM by ×1.3 (30%) and ×1.7 (70%).

Improvement Index Comparison

Bob had the largest gain in performance with a ×6.48 (548% improvement) and ×4.86 (386% improvement) when applying the improvement index metric to compare Intervention 1 and Intervention 2 to baseline, respectively. Steph also produced large gains, with ×2.1 (110% improvement) and ×2.3 (130% improvement) increases in her improvement index compared to her baseline performance. Grace showed an insignificant increase in her improvement index from baseline to Intervention 1 (×1.1,10% improvement) but yielded a robust increase from baseline to Intervention 2 (×2.1, 110% improvement). Luno posted his most significant performance during baseline (×4.5, 350% improvement) anddecreased his improvement index for from baseline to Intervention 1(÷1.3, 23% worsening) and baseline to Intervention 2 (÷1.5, 33% worsening).

Discussion

Successful participation in high school algebra and later achievement require computational fluency and procedural fluency (NMAP, 2008). Given the success of self-managed practice interventions used to build fluency with simple computation (e.g., Hulac et al., 2013;Skinner et al., 1989), the present experiment tested whether students could self-manage a pre-algebra practice intervention to build behavioral fluency with complex computation. The experiment also examined whether differences in fluency performance outcomes would vary between a condition with three 1-min practice trials and a condition with one 3-min practice trial. The intervention included a mnemonic cue, timed practice, and self-directed feedback delivered through answer keys. Visual and quantitative analysis of the SCC served as the evaluative tools.

The researchers used an alternating-treatments design that counterbalanced a baseline condition with two frequency-building (i.e., systematic practice) conditions (Ledford et al., 2019; Sindelar et al., 1985). Evidence suggests Bob's, Steph's, and Grace's performance favored the frequency-building conditions over the baseline condition. Error analysis further indicates that the self-managed feedback component between timings played a significant role in improved accuracy and overall performance. For instance, Bob and Steph, who struggled with element skills in the baseline condition, continued to struggle with an increase in IDSM for the remainder of the condition at ×3.9 per week (290% weekly increase) and ×2.4 per week (140% weekly increase), respectively. Conversely in the experimental conditions, Bob had a ÷6.6 per week and ÷4.7 per week decay in IDSM for Intervention 1 and Intervention 2, respectively, when compared to baseline, as indicated by the celeration multiplier for IDSM. Using the same comparison, Steph had a ÷2.2 per week and ÷2.4 per week decay for Intervention 1 and Intervention 2, respectively.

Grace entered the intervention highly accurate, and that may have plausibly factored into her significant acceleration in CDSM across all three conditions. In addition to a low level of one IDSM across all conditions, she did not produce an IDSM over Days 7–10. When applying the celeration multiplier to compare performance for CDSM, Grace showed a significant improvement over baseline in the Intervention 1 (×1.2, 20% celeration increase) and Intervention 2 (×1.5, 50% celeration increase) conditions. Thus, the additional practice occurring in the intervention conditions likely contributed to her improved speed when compared to the baseline condition.

Students often develop unique error detection skills that lead to managing their own self-feedback to reach a goal (Hattie & Timperley, 2007). In addition, an alternating-treatments design does not always generate a prompt, robust change in performance due to intrasubject variability and prior learning history (Sindelar, et al., 1985). To illustrate, Luno struggled over the first 3 days of intervention as a consequence of solving problems subvocally versus showing his work on paper. He expressed frustration on Day 3 and asked whether showing his work could improve his performance. The researcher reiterated the portion of the directions requesting students show all of their work. On Day 4, Luno's IDSM started to decay significantly, producing the steepest deceleration of IDSM of all the participants by ÷3.0 per week (baseline), ÷2.4 per week (Intervention 1), and ÷1.8 per week (Intervention 2). His delayed response to the intervention protocol plausibly inflated incorrect responding that later confounded the comparison analysis between conditions. However, Luno's profile did provide meaningful information to suggest student performance has different contingencies associated with prior reinforcement history, such as the pervasive mathematics problem-solving issue of "being stubborn" and not "showing their work." By having Luno self-manage and control the process, he came to his own conclusion that his performance suffered from not emitting the fully worked solution.

Unlike the results from Brady and Kubina (2010), where students performed best in three 20-s practice trials versus one 1-min practice trial, the present investigation could not determine an advantage of using one 3-min practice trial versus three 1-min practice trials. Yet, a student's level of accuracy can plausibly dictate the timing that best suits them. For instance, Grace exhibited a robust increase in CDSM in Intervention 2 (×1.9 per week). Because Grace may have required fewer opportunities for corrective feedback, the momentum from uninterrupted practice that occurred during the

3-min practice trial over the three 1-min practice trials may have translated into a stronger performance (Nevin, 1992). Conversely, Bob and Luno showed the largest decrease in celeration for IDSM of ÷1.7 per week and ÷2.4 per week from baseline to Intervention 1, respectively, suggesting that the students may have plausibly benefited from more opportunities for feedback afforded in the condition with three 1-min timings. Increasing the number of feedback opportunities provides more occasions to self-evaluate performance and increases the chances of detecting errors and establishing strategies for future problem solving (Burns et al., 2008; Hattie & Timperley, 2007). All four students exhibited more detail and organization when comparing the practice sheets and assessments from the first day to the last day.

Practical Implications

The frequency-building intervention allowed students to self-manage feedback to execute quick improvements without the type of mediation that can often delay learningand increase the workload of the teacher (Hughes et al., 1991; Mace et al., 2001; McDougall & Brady, 1998; Reid et al., 2005). To organize independent practice effectively, teachers can assign the self-managed intervention to individuals or pairs of students who have already demonstrated accuracy and only need support from procedural cue cards and fully worked solutions. As students work independently, teachers can allocate instructional time and resources toward intensive one-on-one or small-group settings. In the present study, the participants did not receive feedback in the baseline condition, thus restricting the evaluation and understanding of correct or incorrect responses (Burns et al., 2008). In the natural learning environment, the same participants would have likely accelerated in performance from momentary feedback delivered by a teacher versus receiving intensive instruction.

Students who lack fluency in element skills often experience difficulty combining element skills into a compound skill (Binder, 1996; Datchuk, 2016; Johnson & Street, 2013; Kubina et al., 2004). The present intervention effectively exposed and, in some instances, immediately ameliorated error patterns as a result of the daily timed-practice opportunities and self-managed feedback. However, predictable error patterns that occur in compound problem-solving practice activities that do not respond to more immediate feedback often require separate frequency-building activities for element skills (Beverly et al., 2009). For example, Bob finger-counted throughout all three conditions. Finger-counting prompted Bob to pause intermittently, and as a result, he exhibited lower levels of CDSM when compared to the other students. Nonfluency with simple computation serves as an example where a commitment to programmatic change diverts resources to earlier standards (e.g., element skills) but pays large dividends in long-term achievement with complex computations.

Social Validity

The students completed a questionnaire on the last day of the experiment. Three of the four students (Grace, Luno, and Steph) preferred the three 1-min practice trials because they had additional opportunities to correct mistakes on the same problems during the next timing(s). Bob preferred the one 3-min practice trial because he found the length of the task more challenging. All four students found the practice useful and would participate in the intervention as a classroom activity. Grace found decimals (Intervention 2) the most difficult, which reflected her lowest output of CDSM (geometric mean of 44.27). Yet, Grace produced her highest performance growth of ×3.4 (240% weekly change rate) during Intervention 2. Other comments made by the students included "it increases the amount of thought you put in", "it helps your math skills and since math classes are about learning new concepts","they won't have to think about simple facts as much", and "I think daily practice like this would help students fully understand order of operations a lot better."

Limitations

An alternating-treatments design can lead to treatment interference across conditions. In the current scenario, the PEMDAS mnemonic and repeated practice to build speed likely diffused between conditions, as did prior knowledge of skills inherent in intrastudent variability. Because element skills such as simple computation and knowledge of procedures can impact students' performance of complex computation, future researchers may want to conduct component analyses to determine whether fluency-building activities should occur prior to the study. Yet, the present study may accurately reflect the state of fluency instruction in schools and the type of outcomes that would typically occur in general education settings. We had a small window of time during the first 15–20 min of math instruction to test the research questions, and to request a focus on simple computation prior to complex computation would have conflicted with instructional programming and stymied the investigation. Perhaps time constraints and the relative absence of any type of meaningful practice activities to support element skills may also serve as the reasons for the paucity of research associated with prealgebraic fluency. The present experiment serves as one of two initial investigations into frequency building with prealgebra concepts (Stocker et al., 2018).

Future Directions for Research

Behavioral fluency functions as a key component in the hierarchical configuration of mathematics curricula and standards. Achieving behavioral fluency efficiently and effectively requires timed repetition of behavior and subsequent feedback given at the end of the timed trial (Hughes et al., 2007; Kubina, 2019). Students who reach a fluent level of performance or fluency aim with an element skill exhibit critical learning outcomes such as long-term retention, endurance in the presence of distraction, and application to a new compound behavior (Kubina & Yurich, 2012; McTiernan et al., 2016). Future studies that focus on critical learning outcomes that result from frequency building and prealgebraic computations would provide a new direction of research, as well as complement the growing behavioral-fluency research literature.

In the present investigation, students who entered the study with a stronger grasp of element skills (e.g., simple computation, fractions, decimals) performed more fluidly within the context of order of operations. In the future, order of operations will serve as an element skill for more complex algebraic processes. Attaining a level of fluency with element skills has the potential to positively impact performance in algebra. Replication of the present study can provide evidence and incentive for schools to focus on intervention packages that include a focus on both element and compound skills.

A paucity of frequency-building and other fluency-building interventions exists for pre-algebra. Future studies should therefore continue to compare student performance using different timed conditions, problem types, and experimental designs. Additional studies can also lead to findings to support different student populations (e.g., special education) and increase the research base to support fluency instruction in middle school mathematics.

Compliance with Ethical Standards

Conflict of interest James D. Stocker declares no conflict of interest. Richard M. Kubina owns equity in CentralReach. The financial interest has been reviewed by Pennsylvania State University's Individual Conflict of Interest Committee and is currently being managed by the University.

References

Adelman, C. (2006). *The toolbox revisited: Paths to degree completion from high school through college*. Department of Education: U.S.

Archer, A., & Hughes, C. (2011). *Explicit instruction: Efficient and effective teaching*. Guilford Publications.

Ardoin, S. P., & Daly, E. J. (2007). Introduction to the Special Series: Close encounters of the instructional kind: How the instructional hierarchy is shaping instructional research 30 years later. *Journal of Behavioral Education, 16*, 1–6. https://doi.org/10.1007/s10864-006-9027-5.

Beverly, M., Hughes, J. C., & Hastings, R. P. (2009). What's the probability of that? Using SAFMEDS to increase undergraduate success with statistical concepts. *European Journal of Behavior Analysis, 10*, 183–195. https://doi.org/10.1080/15021149.2009.11434321.

Beverly, M., Hughes, J. C., & Hastings, R. P. (2016). Using SAFMEDS to assist language learners to acquire second-language vocabulary. *European Journal of Behavior Analysis, 17*, 131–141. https://doi.org/10.1080/15021149.2016.1247577.

Binder, C. (1996). Behavioral fluency: Evolution of a new paradigm. *The Behavior Analyst, 19,* 163–197. https://doi.org/10.1007/BF03393163.

Binder, C. (2003). Doesn't everybody need fluency? *Performance Improvement, 42,* 14–20. https://doi.org/10.1002/pfi.4930420304.

Brady, K. K., & Kubina, R. M. (2010). Endurance of multiplication fact fluency for students with attention deficit hyperactivity disorder. *Behavior Modification, 34*, 79–93. https://doi.org/10.1177/0145445510361331.

Bullara, D. T., Kimball, J. W., & Cooper, J. O. (1993). An assessment of beginning addition skills following three months without instruction or practice. *Journal of Precision Teaching, 11*, 11–16.

Burns, M. K., VanDerHeyden, A. M., & Boice, C. H. (2008). Best practices in delivery of intensive academic interventions. In A. Thomas & J. Grimes (Eds.), *Best practices in school psychology* (5th ed., pp. 1151–1162). National Association of School Psychologists.

CentralReach. (2019). PrecisionX [Computer software]. https://centralreach.com/

Chiesa, M., & Robertson, A. (2000). Precision teaching and fluency training: Making maths easier for pupils and teachers. *Educational Psychology in Practice, 16*, 297–310. https://doi.org/10.1080/02667360020006372.

Clark-Carter, D. (2005). Geometric mean. In B. Everitt & D. Howell (Eds.), *Encyclopedia of statistics in behavioral science* (pp. 744–745). John Wiley & Sons. https://doi.org/10.1002/0470013192.bsa376

Codding, R. S., Burns, M. K., & Lukito, G. (2011). Meta-analysis of mathematic basic-fact fluency interventions: A component analysis. *Learning Disabilities Research & Practice, 26*, 36–47. https://doi.org/10.1111/j.1540-5826.2010.00323.x.

Common Core State Standards Initiative. (2010). *Common Core State Standards for mathematics.* http://www.corestandards.org/assets/CCSSI_Math%20Standards.pdf

Common Core State Standards Initiative. (2018). Common core state standards for mathematics 2010. Common Core State Standards Initiative.

Cooper, J. O., Heron, T. E., & Heward, W. L. (2020). *Applied behavior analysis* (3rd ed.). Pearson.

Daly, E. J., III., Martens, B. K., Barnett, D., Witt, J. C., & Olson, S. C. (2007). Varying intervention delivery in response to intervention: Confronting and resolving challenges with measurement, instruction, and intensity. *School Psychology Review, 36,* 562–581. https://doi.org/10.1080/02796015.2007.12087918.

Datchuk, S. M. (2016). Writing simple sentences and descriptive paragraphs: Effects of an intervention on adolescents with writing difficulties. *Journal of Behavioral Education, 25*, 166–188. https://doi.org/10.1007/s10864-015-9236-x.

Deno, S. L., & Mirkin, P. K. (1977). *Data-based program modification: A manual*. Reston VA: Council for Exceptional Children.

Fitzgerald, D. L., & Garcia, H. I. (2006). Precision teaching in developmental mathematics: Accelerating basic skills. *Journal of Precision Teaching and Celeration, 22*, 11–28.

Foegen, A., Olson, J. R., & Impecoven-Lind, L. (2008). Developing progress monitoring measures for secondary mathematics: An illustration in algebra. *Assessment for Effective Intervention, 33*, 240–249. https://doi.org/10.1177/1534508407313489.

Fuchs, L. S., Seethaler, P. M., Powell, S. R., Fuchs, D., Hamlett, C. L., & Fletcher, J. M. (2008). Effects of preventative tutoring on the mathematical problem solving of third-grade students with math and reading difficulties. *Exceptional Children, 74*, 155–173. https://doi.org/10.1177/001440290807400202.

Geary, D. C., Hoard, M. K., Byrd-Craven, J., Nugent, L., & Numtee, C. (2007). Cognitive mechanisms underlying achievement deficits in children with mathematical learning disability. *Child Development, 78*, 1343–1359. https://doi.org/10.1111/j.1467-8624.2007.01069.x.

Hattie, J., & Timperley, H. (2007). The power of feedback. *Review of Educational Research, 77*, 81–112. https://doi.org/10.3102/003465430298487.

Hughes, C. A., Korinek, L., & Gorman, J. (1991). Self-management for students with mental retardation in public school settings: A research review. *Education & Training in Mental Retardation, 26*, 271–291.

Hulac, D. M., Wickerd, G., & Vining, O. (2013). Allowing students to administer their own interventions: An application of the self-administered folding-in technique. *Rural Special Education Quarterly, 32,* 31–36. https://doi.org/10.1177/875687051303200206.

Johnson, K., & Street, E. M. (2013). *Response to intervention with precision teaching: Creating synergy in the classroom*. Guilford. https://doi.org/10.1353/etc.2014.0010.

Johnston, J. M., & Pennypacker, H. S. (2009). *Strategies and tactics of behavioral research* (3rd ed.). Routledge. https://doi.org/10.4324/9780203837900.

Kazdin, A. E. (2011). *Single-case research designs: Methods for clinical and applied settings* (2nd ed.). Oxford University Press.

Kostewicz, D. E., King, S. A., Datchuk, S. M., Brennan, K. M., & Casey, S. D. (2016). Data collection and measurement assessment in behavioral research: 1958–2013. *Behavior Analysis Research and Practice, 16*, 19–33. https://doi.org/10.1037/bar0000031.

Kubina, R. M. (2019). *Precision teaching implementation manual*. Greatness Achieved Publishing.

Kubina, R. M., Young, A. E., & Kilwein, M. (2004). Examining an effect of fluency: Application of oral word segmentation and letters sounds for spelling. *Learning Disabilities: A Multidisciplinary Journal, 13*, 17–23.

Kubina, R. M., & Yurich, K. K. L. (2012). *Precision teaching book*. Greatness Achieved Publishing.

Ledford, J. R., & Gast, D. L. (Eds.). (2009). Single subject research methodology in behavioral sciences: applications in special education and behavioral sciences. Routledge. https://doi.org/10.4324/9780203877937

Ledford, J. R., Barton, E. E., Severini, K. E., & Zimmerman, K. N. (2019). A primer on single-case research design: Contemporary use and analysis. *American Journal on Intellectual and Developmental Disabilities, 124,* 35–56. https://doi.org/10.1352/1944-7558-124.1.35.

Maccini, P., Mulcahy, C. A., & Wilson, M. G. (2007). A follow-up of mathematics interventions for secondary students with learning disabilities. *Learning Disabilities Research & Practice, 22*, 58–74. https://doi.org/10.1111/j.1540-5826.2007.00231.x.

MacDonald, J. E., Wilder, D. A., & Binder, C. (2006). The use of precision teaching techniques to increase mathematics skills in adults with schizophrenia. *Journal of Precision Teaching and Celeration, 22*, 2–10.

Mace, E C., Belfiore, P. J., & Hutchinson, J. M. (2001). Operant theory and research on self-regulation. In B. Zimmerman & D. Schunk (Eds.), *Self-regulated learning and academic achievement* (pp. 39–66). Lawrence Erlbaum.

Manalo, E., Bunnell, J. K., & Stillman, J. A. (2000). The use of process mnemonics in teaching students with mathematics learning disabilities. *Learning Disability Quarterly, 23*, 137–156. https://doi.org/10.2307/1511142.

Mastropieri, M. A., & Scruggs, T. E. (1989). Constructing meaningful relationships: Mnemonic instruction for special populations. *Educational Psychology Review, 1*, 83–111. https://doi.org/10.1007/BF01326638.

Mastropieri, M. A., & Scruggs, T. E. (1998). Enhancing school success with mnemonic strategies. *Intervention in School and Clinic, 33*, 201–208. https://doi.org/10.1177/105345129803300402.

McDougall, D., & Brady, M. P. (1998). Initiating and fading self-management interventions to increase math fluency in general education classes. *Exceptional Children, 64*, 151–166. https://doi.org/10.1177/001440299806400201.

McFarland, J., Hussar, B., Wang, X., Zhang, J., Wang, K., Rathbun, A., Barmer, A., Forrest Cataldi, E., and Bullock Mann, F. (2018). The Condition of Education 2018 (NCES 2018-144). U.S. Department of Education. Washington, DC: National Center for Education Statistics. Retrieved [date] from https://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=2018144.

McTiernan, A., Holloway, J., Healy, O., & Hogan, M. (2016). A randomized controlled trial of the Morningside math facts curriculum on fluency, stability, endurance, and application outcomes. *Journal of Behavioral Education, 25*, 49–68. https://doi.org/10.1007/s10864-015-9227y.

Miller, A. D., Hall, S. W., & Heward, W. L. (1995). Effects of sequential 1-minute time trials with and without inter-trial feedback and self-correction on general and special education students' fluency with math facts. *Journal of Behavioral Education, 5*, 319–345. https://doi.org/10.1007/BF02110318.

National Academies of Sciences, Engineering, and Medicine. (2017). *Building America's skilled technical workforce*. The National Academies Press. https://doi.org/10.17226/23472

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.*

National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. U.S. Department of Education.

Nelson, P. M., Burns, M. K., Kanive, R., & Ysseldyke, J. E. (2013). Comparison of a math fact rehearsal and a mnemonic strategy approach for improving math fact fluency. *Journal of School Psychology, 51*, 659–667. https://doi.org/10.1016/j.jsp.2013.08.003.

Nevin, J. A. (1992). An integrative model for the study of behavior momentum. *Journal of the Experimental Analysis of Behavior, 57,* 301–316.

Reid, R., Trout, A. L., & Schartz, M. (2005). Self-regulation interventions for children with attention deficit/hyperactivity disorder. *Exceptional Children, 71*, 361–377.

Rittle-Johnson, B., & Siegler, R. S. (1998). The relation between conceptual and procedural knowledge in learning mathematics: A review. In C. Donlan (Ed.), *The development of mathematical skills* (pp. 75–110). Psychology Press.

Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology, 93*, 346-362. https://doi.org/10.1037/0022-0663.93.2.346.

Rivera, D. M., & Bryant, B. R. (1992). Mathematics instruction for students with special needs. *Intervention in School and Clinic, 28*, 71–86. https://doi.org/10.1177/105345129202800203.

Sindelar, P. T., Rosenberg, M. S., & Wilson, R. J. (1985). An adapted alternating treatments design for instructional research. *Education and Treatment of Children, 8*, 67–76.

Stocker Jr., J. D., Kubina, R. M., Riccomini, P. J., & Mason, A. (2018). Comparing the effects of different timings to build computational and procedural fluency with complex computation. *Journal of Evidence-Based Practices for Schools, 16*, 206–231.

Stromgren, B., Berg-Mortensen, C., & Tangen, L. (2014). The use of precision teaching to teach basic math facts. *European Journal of Behavior Analysis, 15,* 225–240. https://doi.org/10.1080/15021149.2014.11434723.

Wang, X. (2013). Why students choose STEM majors: Motivation, high school learning, and postsecondary context of support. *American Educational Research Journal*, *50*, 1081–1121. https://doi.org/10.3102/0002831213488622.

Witzel, B. S., & Riccomini, P. J. (2007). Optimizing math curriculum to meet the learning needs of students. *Preventing School Failure: Alternative Education for Children and Youth, 52*, 13–18. https://doi.org/10.3200/PSFL.52.1.13-18.

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.