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A Preliminary Investigation of the Relationship Between Fluency and Application for Multiplication

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Research suggests component skill performance has a strong positive relationship with composite skill performance. This study examined the association between accuracy and fluency for the component-composite relationship within multiplication. One hundred and fifty-seven fifth-graders did one-minute assessments for single-digit, and multi-digit multiplication problems. The results demonstrated the students achieved high levels of accuracy but low levels of fluency. Strong correlations between the component-composite skill fluency suggest that fluent component skills may have a significant role in composite skill performance. Moderate/low correlations between component and composite skill accuracy indicate that more than one skill component may contribute to composite skill acquisition.

KEY WORDS: fluency; application; component and composite performance.

THE RELATIONSHIP BETWEEN FLUENCY AND APPLICATION FOR MULTIPLICATION

Mathematics is one of the primary academic subjects in a public school curriculum. How students later perform mathematically affects labor quality and national growth (Hanushek & Kimko, 2000). A substantial amount of money, time and effort has been invested in mathematic education. For example, the US Department of Education Budget Office (2003) estimated it would spend $1 billion dollars over five years for requested mathematics and science partnerships. Data, unfortunately, suggests American students are not performing as well as they could especially in light of the tremendous amount of money spent on education.

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Indicators of American students' math performance come from tests like the National Assessment of Educational Progress (NAEP). The NAEP is a nationally representative, ongoing assessment of American student academic performance which examines the math performance of students in grades 4, 8, and 12. The last NAEP report showed that less than 30% of students across the selected grades achieved proficiency in mathematics performance (National Center for Education Statistics, 2000). International mathematic assessments like the Trends in International Mathematics and Science Study or TIMSS (1999) also show that the American students' average achievement in various math content areas are commensurate with countries such as Romania, Spain, and Iran; Countries who do not spend billons of dollars on education.

A number of theories have been proposed to explain why American students do not perform better on national and international tests. Some reasons for students' poor performance are attributed to student demographic background (Zhang & Zhang, 2002), teacher certification and training (New York City Board of Education, 2000a), academic allotted time (New York City Board of Education, 2000b), student self-concept and motivation (Lorenz, 1982), and a variety of other factors. While any of the previously mentioned theories may account for the students' performance, behavioral research examines behavior in a direct fashion and offers a number of alternative explanations for deficits in math. Namely, how well students master foundational skills impacts the acquisition of subsequent skills (Binder, 1996; Bucklin, Dickinson, & Brethower, 2000; Haughton, 1972; Mercer, Campbell, Miller, Mercer, & Lane, 2000). Stated differently, directly examining foundational skills of mathematic performance may show a significant impact on later skill acquisition as opposed to analyzing indirect variables (e.g., self-concept, motivation).

Shapiro (1996) noted that students in special education and beyond have difficulty mastering basic skills which can influence attainment of subsequent advanced skills. This relationship encompasses not only mathematics but also any other basic skill plays a pivotal role in an advanced skill. For example, concert violinists learn basic fingering positions before playing complex pieces. Opera singers first master high notes before singing arias. And students solving complex Algebra II problems must first become fluent with basic addition and multiplication facts (Wu, 1999). The term for a direct analysis of basic and advanced skills is "application." Application refers to instances where a student "applies" basic or component skills to more advanced or composite skills (Binder, 1996; Johnson & Layng, 1992; McDowell & Keenan, 2001).

For application, basic or foundational skills are called components or elements and they combine to form more complex responses termed composites or compounds (Barrett, 1979; Binder, 1996; Eaton, 1978; Johnson & Layng, 1992, 1994). A composite skill may be the melding of two or more component skills in planned or novel circumstances. For instance, drawing lines and circles are component skills to the composite skill writing alphabet letters (Zaner-Bloser, 1999). Writing alphabet letters, however, is a component skill to writing words, the composite skill in this pair. In general, component skills have to be learned prior to the instruction of composite skill so that students may readily acquire the more advanced composite skill (e.g., Binder, 1996; Carnine, Silbert, Kameenui, & Tarver, 2004; Christenson, Ysseldyke, & Thurlow, 1989; Mercer & Mercer, 2001; Lovitt, 1978; Rosenshine & Stevens, 1986; Stein, Silbert, & Carnine, 1997).

Application studies demonstrate how fluency with component skills affect composite skill performance (Berens, Boyce, Berens, Doney, & Kenzer, 2003; Kubina, Young, & Kilwein, 2004; McDowell & Keenan, 2002; McDowell, Keenan, & Kerr, 2002; McDowell, McIntyre, Bones, & Keenan, 2002; McIntyre, Test, Cooke, & Beattie, 1991; Smyth & Keenan, 2002). For example, Bucklin et al. (2000) conducted a basic research study examining the application of Hebrew symbols and arithmetic tasks with 30-college students. The participating students had no prior knowledge of Hebrew symbols. They were divided into two groups. Component tasks were then introduced to each group; (1) Hebrew symbols associated with syllables and (2) syllables associated with Arabic numerals. One group practiced the component skills until they demonstrated mastery or 100% accuracy in each task. The other group continued practicing until their performance reached fluency, or a combination of speed and accuracy. Neither group received formal instruction in the composite skill which was applying Hebrew symbols to associated Arabic numerals in arithmetic tasks. After the students achieved their respective aims, they were asked to perform the composite skill. The result illustrated that students who achieved fluency performed the composite skill significantly better than those who simply achieved accuracy alone. This study demonstrated that application could occur when individuals were fluent in component skills even without formal instruction with the composite skill.

Application research such as that conducted by Bucklin et al. (2000) emphasize the relationship between achieving fluency with component skills and the subsequent acquisition of the composite skill. Other studies suggest application can be studied with mathematic skills (e.g., Berens, Boyce, Berens, Doney, & Kenzer, 2003; McDowell & Keenan, 2002; McDowell, Keenan, & Kerr, 2002; McIntyre, Test, Cooke, & Beattie, 1991). The previously cited studies targeted participants who had difficulties with advanced skills and conducted an intervention in their basic skill. Current research does not demonstrate what relationship exists between students who may or may not have difficulties with advanced mathematic skills. If students were fluent with the component skills of mathematics would higher levels of those skills correlate with a higher level of a composite skill? Also, does the dominant method of teaching to "mastery," or more specifically termed accuracy, in component skills contribute to improvement of composite skill performance?

To answer questions investigating the relationship between achieving accuracy and fluency with component skills the current study examined a primary competency taught in elementary mathematic curricula, multiplication (Stein et al., 1997). The components and composites of multiplication can be clearly identified in terms of varying levels of complexity. Therefore, the experimenters investigated the relationship between the performance of a component skill, single-digit multiplication, and a composite skill, multi-digit multiplication. The research questions asked in this study follow: (1) How well will the participating 5th grade students perform basic multiplication facts and multi-digit multiplication problems in terms of accuracy and fluency; and (2) What is the relationship between basic multiplication fact performance and multi-digit problem completion.

METHOD

Participants

Five public elementary schools from three school districts in central and southeastern Pennsylvania participated in the study. After receiving permission from the district to conduct the study, principals from each school recruited fifth-grade students' participation and obtained parental consent. Principals gave parental consent forms to all of the students in their respective 5th grade classes. Students who returned signed consent forms from their parents and who additionally signed a consent form themselves participated in the study.

Included in the study were one urban, three suburban, and one rural school. Students classified as "low-income" (Pennsylvania Department of Education, 2004) in the five schools ranged from 10% to 45%. Of the students who participated, 74 were male and 83 female. The total sample represented 43% of the fifth graders served by the five schools. This study did not record students' demographic backgrounds because of a confidentiality requirement, however the principals reported approximately 10% of the participating students were enrolled in learning support classes or receiving special education services.

Materials

A test packet consisting of three sections was constructed. The first section, a screening measure, used lined notebook paper so students could repeatedly write Arabic numerals from 0 to 9. The second section assessed student component skill performance by presenting 156 random single digit multiplication problems. The third section evaluated composite skill performance. A two-page composite assessment sheet had 63 random multi-digit multiplication questions ranging from 2 or 3 digits multiplicands by 1 or 2 multipliers with an even distribution of problems with and without renaming/regrouping. Both the second and third section were generated with Microsoft Excel® using the "Rand" function. Copies of each assessment are available upon request. An electronic countdown timer was also used to begin and end the assessment intervals.

Procedure

Participating students in each individual school came down as a group to an empty classroom or cafeteria. The experimenter briefly described the purpose of the study. Students were informed that they should answer each of the three test sections in one-minute as quickly and accurately as possible. The one-minute assessment procedure came from a method described by Van Houten and Thompson (1976) as an "explicit timing procedure for math performance." An electronic countdown timer was presented and the timing procedure was demonstrated before the test. The experimenter used the following scripted message:

> You are going to do three short tests. Each test lasts one-minute. I will use this timer to set one-minute. For each test, when I say "ready . . . go," please start the test. When the timer starts to beep like this, please stop and put your pencil on the table.

The experimenter distributed test packets to each student and directed them not to turn the page until instructed to do so. Testing instructions were provided before each test.

The first section measured students' numeral writing speed. Students were informed to repeatedly write 0 to 9 on notebook lined paper. The experimenter demonstrated a left to right writing motion and answered questions before beginning the timing. In the second and third sections of the tests, basic and advanced multiplication questions were presented. Students were told to cross their answers out when they made a mistake instead of using erasers, and to skip questions if they did not know how to complete the problem. Each of the multiplication assessment sheets contained two pages. Students were told and shown to start from the first page of the section and continue working on the second page if they finished the first one.

The measurement unit of this study was a correctly written digit. Students were requested to write down the completed algorithms of their multi-digit multiplication problems in the third section in addition to the final product. Each digit in an algorithm was counted toward students' final completed digits and/or correct digits. For example, the multiplication problem of 261 times 52 includes an algorithm of 522 plus 13050 equal to 13572, which yielded the total of 13 correct digits. If a student only wrote 522, three correct digits were counted toward his/her total skill performance in this section even though the final correct answer 13572 was absent. Students were reminded that the timing was only an assessment and there were more questions than they could finish in one-minute. Each section of the test lasted one-minute and was conducted once for each individual student.

Dependent Variables

Completed test packets were compared with a pre-constructed key. The total correct digits were counted to measure two variables—fluency and accuracy. The fluency variable was measured as the number of correct digits per minute regardless of the total completed digits and errors. The accuracy variable referred to the percentage of correct digits and total completed digits, specifically the correct digits/total completed digits multiplied by 100. Four variables were analyzed: component skill fluency, component skill accuracy, composite skill fluency, and composite skill accuracy.

Descriptive statistics were used to examine the data to answer the first research question. How would elementary students' basic and advanced multiplication performance compare with suggested performance standards or fluency aims (Johnson, 1996; Kubina & Lin, 2003; Mercer, Mercer, & Evans, 1982; Wood, Burke, Kunzelmann, & Koenig, 1978). Accuracy standards for both component and composite skills were 100% correct. This experiment used 100% correct as a measure of mastery because 100% represented the ultimate terminal criterion for an accuracy measure. The fluency aim for single digit multiplication was 80 to 120 correct digits per minute and 40 to 60 correct digits per minute for multiple digit multiplication. The fluency aims came from the Precision Teaching literature. The Precision Teaching literature suggests that high aims correlate with advanced composite skill performance (Koorland, Keel, & Ueberhorst, 1990). Koorland et al. also point out that more research is needed to experimentally verify fluency aims, however the descriptive data does suggest the fluency aims for math indicate fluent performance.

Pearson correlation coefficients were conducted to analyze relationships among the variables. Theoretically one could not produce more correct digits in any type of mathematics questions than the maximum numerals he/she was able to write in same period of time. Numeral writing speed served as a protection from underestimating multiplication performance of those students who might have slow handwriting. If a student's numeral writing performance was lower than the fluency aim of multi-digit multiplication, 40 digits per minute, it was not included in the correlational analysis.

Interscorer Agreement

An independent scorer counted correct and total digits every tenth test packet across three test sections. Agreement was calculated by dividing the smaller total by the larger total of counted digits. The mean of interscorer agreement across three sets was 99.5% ranging from 86%–100%. The mean for numeral writing was 99.9% (range 97.9%–100%), component skill was 99.2% (range 93.8–100%), and composite skill was 99% (range 86%–100%).

Table I. Mean, Range, and Standard Deviation Comparison

Variable	*n*	Range minimum	Range maximum	*M*	*SD*
Numeral writing	156	30	138	91.85	18.35
Component fluency (digit)	156	16	112	57.01	19.48
Component accuracy (percent)	156	78	100	98.12	3.75
Composite fluency (digit)	156	1	56	18.71	11.01
Composite accuracy (percent)	156	23	100	88.40	16.63

RESULTS

Of the 157 completed packets, one was completed incorrectly and discarded. Thus, a total of 156 packets were analyzed. The first research question asked how well would the participating 5th grade students perform basic multiplication facts and multi-digit multiplication problem in terms of accuracy and fluency. The mean numeral writing frequency was 91.85 digits per minute ranging between 30 and 138 as shown in Table I. One student wrote less than 40 digits per minute, Table II, and this pack was excluded from the correlation analysis.

The component skill fluency mean, or average correct digits per minute for single-digit multiplication problems, shown in Table I were 57.01 digits per minute ranging between 16 and 112. Fourteen percent of the students performed at the fluency aim, Table II. The component skill accuracy was high ($M = 98.12\%$ correct) and had little variance ($SD = 3.75$). The component skill data indicated that 64% of the fifth graders in this study had met the accuracy standard of 100% in single-digit multiplication.

Table II. Comparison with Performance Aims

Variables	Frequency	Percent (%)	Mean	Std. deviation
Fluency aim (used composite skill aim) = 40 correct digits per minute				
Numeral writing < aim	1	0.6	30	—
Numeral writing =, > aim	155	99.4	92.25	17.67
Fluency aim = 80 correct digits per minute				
Component < aim	134	85.9	51.25	13.91
Component =, > aim	22	13.1	92.09	8.71
Fluency aim = 40 correct digits per minute				
Composite < aim	151	96.8	17.79	9.87
Composite =, > aim	5	3.2	46.4	5.16
Accuracy aim = 100% correct				
Component < aim	56	35.9	95	5
Component = aim	100	64.1	—	—
Accuracy aim = 100% correct				
Composite < aim	81	51.9	78	17
Composite = aim	75	48.1	—	—

Table III. Variability among Accuracy and Fluency

Accuracy condition	Range minimum	Range maximum	Mean	Std. deviation
Component fluency				
Component accuracy below 100% accuracy	16	110	55.5	22.02
Component accuracy equal to 100% accuracy	19	112	58.11	17.65
Composite fluency				
Composite accuracy below 100% accuracy	1	47	19.51	10.02
Composite accuracy equal to 100% accuracy	3	56	18.14	11.73

The composite skill fluency mean, Table I, multi-digit multiplication problems was relatively low ($M = 18.71$ digits per minute). Only 3.2% of the students achieved the fluency aim of 40 digits per minute shown in Table II. The composite skill accuracy was moderately high ($M = 88.40$% correct; 51.9% students achieved 100% correct) with a large variance ($SD = 16.63$; range 23 to 100).

When accuracy levels were held consistent, 100% accuracy, both component and composite fluency exhibited great variances (component fluency $SD = 17.65$; composite fluency $SD = 11.73$). The variances were similar to those when accuracy levels were not consistent or below 100% accuracy (component fluency $SD = 22.02$; composite fluency $SD = 10.02$). The accuracy level in each skill was not positively associated with fluency level (see Table III). In summary, most students achieved high levels of accuracy in both multiplication problems. However, few students reached the fluency aim in either skill. Variability was greater in composite skill than in component skill.

The second research question examined the relationship between component and composite skill performance for the two math tasks. The total sample number for this correlation analysis was 155. As mentioned previously, one test packet was excluded because the student's numeral writing frequency did not reach the minimum digits per minute. Table IV shows significant correlations among four variables, component fluency and accuracy, composite fluency and accuracy. Component fluency was most strongly associated with composite fluency ($r = .745$, $\rho < .01$) but had low relationship with component accuracy ($r = .252$,

Table IV. Intercorrelations Between Fluency and Accuracy

Variables	Composite fluency	Composite accuracy
Component fluency	.745**	.252**
Component accuracy	.191*	.371**

Note. $n = 155$.
$^{*}\rho < .05$ (two-tailed). $^{**}\rho < .01$ (two-tailed).

$\rho < .01$). Component accuracy was moderately related with composite accuracy ($r = .371$, $\rho < .01$) and weakly associated with composite fluency ($r = .191$, $\rho < .05$).

DISCUSSION

This study investigated the relationship between fluency and accuracy for component and composite skill multiplication performance with elementary aged students. As part of this study the data revealed how fifth grade students performed multiplication tasks by measuring accuracy and fluency. Among the students participating in this study, single digit multiplication performance was highest for accuracy, not fluency. Only 13% of the 155 students achieved the fluency aim for basic multiplication facts while 64% of students scored 100% accuracy. Even fewer students achieved the fluency aim with the composite skill; only 3% met the aim. The data points to a critical need for fluency building with multiplication.

The relationship between students' performance for component and composite skills demonstrated a strong correlation in fluency between single-digit and multi-digit multiplication. Component accuracy, on the other hand, had a low correlation with composite accuracy and composite fluency. Considering students' performance in this study, the results suggest that building fluency in single-digit multiplication may play a significant role in accelerating fluency in multi-digit multiplication. However, the weak association between component fluency and composite accuracy did not support the hypothesis. Higher frequencies in fluency of the component skill were not associated with higher levels of accuracy in the composite skill.

General Student Performance

Component Skill Performance

The data provided evidence that students can achieve high accuracy levels with a component skill but have not yet attained fluency for that skill. In other words, the students acquired the single-digit multiplication but did not yet become fluent or what Binder (1996) calls "true mastery." The students' learning was in a proficiency or fluency-building stage (Johnson et al., 1992; Mercer et al., 1993). The fluency stage requires students to be both accurate and fast. Although students may have performed the component skill with high degrees of accuracy, they needed additional practice to reach the fluency aim.

The data also provided support for the need to have students practice to fluency and that accuracy alone is insufficient to differentiate between truly mastered skills (Binder, 1996; Kubina & Morrison, 2000). Two students who had the same accuracy level performed very differently in terms of fluency. For example, one student had 100% accuracy and produced 19 correct digits per minute while another student also had 100% accuracy but produced 112 correct digits per minute. Practicing to fluency and measuring the data with frequency seem to be critical if teachers value distinguishing skilled math performance among students.

Composite Skill Performance

Compared with component skill competence, participants demonstrated relatively low accuracy in the composite skill and showed a high degree of variability. Only half of the students in the sample achieved the accuracy aim, which suggests that not all 5th grade students participating in this study had acquired multi-digit multiplication competency. Insufficient time and instruction with the more complex math tasks would limit students from achieving the accuracy aim. At the time of the study however, the participating 5th grade students should have received instruction with multi-digit multiplication and other advanced multiplication tasks such as two digits by two digits with and without regrouping (Stein et al., 1997). Principals of the students confirmed that multi-digit multiplication was introduced before the study began.

When examining students' fluency with the composite skill, only 3% of the students in the sample achieved the fluency aim. Such data show that fluent performance with multi-digit multiplication appears nearly absent at the 5th grade level with the selected participants. Considering the low level of accuracy, the finding of low fluency performance in the composite skill also supports the proposition that fluency building can be more difficult if acquisition is not first achieved. Rhymer, Skinner, Henington, D'Reaux, and Sims (1998) indicated that targeting fluency without high levels of accuracy increases error rate. Therefore, students trying to become fluent with the composite skill, such as multi-digit multiplication, will encounter a more demanding task if they are not fluent with the correct algorithm for solving the problem in the first place.

Inadequate composite skill performance provided an additional explanation for the higher degree of fluency variability within the same accuracy level. An analysis of the data showed that some students used addition to calculate multi-digit multiplication. For example, in the problem $69 \times 4 =$ ____ a student wrote four sixty-nines and used addition to obtain the final answer. The result was 100% correct, but only the final answer in digits was counted as correct because the successive addition performance was not the correct multiplication algorithm. Successively adding is a time-consuming strategy and negatively impacted the student's ability to answer questions. Additionally, the successive adding strategy becomes very cumbersome, if not impractical, when students encounter problems such as $69 \times 69 =$ ____. Students' lack of the accuracy with the correct multi-digit algorithm produced high accuracy but low fluency and decreased the correlation between these two measurements in composite skills.

Component Skill Fluency for Application

Component fluency was the most powerful indicator for composite fluency. This finding confirmed the previous research of application which stated that component skills, when fluent, are more likely to apply and promote expeditious learning of a composite skill (Binder et al., 1989, 1996; Bucklin et al., 2000; Evans, Mercer, & Evans, 1983). The current data clearly shows that the majority of the participating students did not have component skill fluency and subsequently had even less competence with composite skill fluency. With so few students in the sample accomplishing the component fluency aim and the strong correlation between component and composite skill fluency, the low level of single-digit multiplication fluency might be a causative factor for the poor performance with multi-digit multiplication. Because this study was correlational, future experimental studies may determine the causative effect fluency plays with composite math skills.

Component fluency did not have a strong correlation with composite accuracy. One possible explanation is that fluency might be independent from accuracy in terms of application. Increasing fluency in component skills does not necessarily generate more accuracy in the composite skill, especially if students have not acquired the correct algorithm for the composite skill. Adequate formal instruction in composite skill seems to be essential to promote accuracy in addition to the fluency building for component skills.

Component Skill Accuracy for Application

In contrast to the fluency variables, component skill accuracy was not strongly associated with composite accuracy or with composite fluency. Observations of some student performances, when written by the students in their test packet, in composite algorithms provided possible explanations. The data indicated that some students had trouble with complex addition problems and place value skills. Even with correct single digit multiplication, these students made mistakes with the final product, which decreased their accuracy level. For example, in the problem of $986 \times 83 = \_\_\_\_$ several students had written the correct algorithms, such as $2958 + 78880$, but added incorrectly. Other students made errors in place value like placing the last 8 in the tens column, for 78880, 8 in the ones column with the resulting addition problem of $2958 + 7888 = 10846$.

Single digit multiplication, complex addition and place value are all considered prerequisite skills of multi-digit multiplication (Cordoni, 1987; Kamii

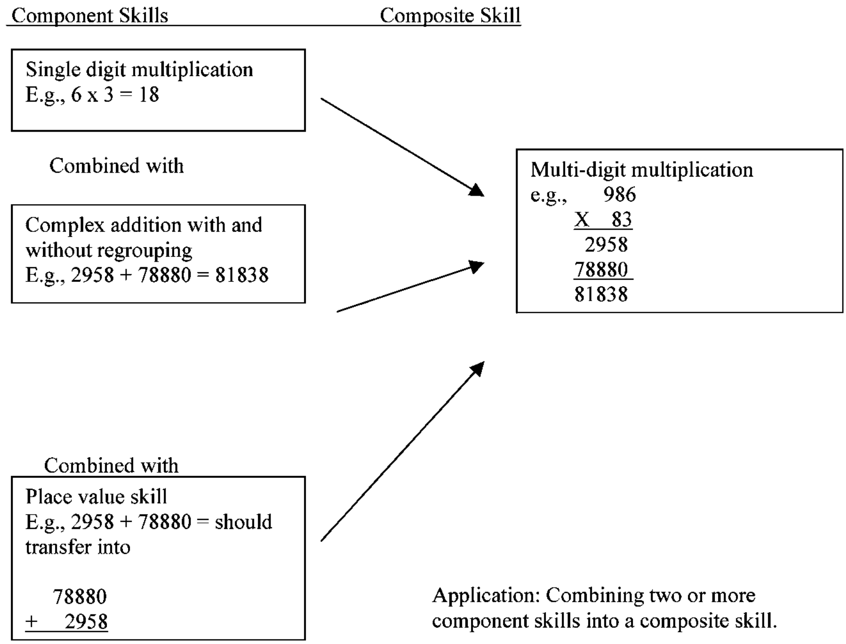

Fig. 1. Component skills for multi-digit multiplication.

& Joseph, 1988; Ott, 1990; Stein et al., 1997; Van de Walle, 1990). If students had problems with the later two component skills, it was understandable that accuracy of single digit multiplication alone was not sufficient to predict either accuracy or fluency of multi-digit multiplication (see Figure 1 for graphic description). Low levels of accuracy in complex addition and place value skills might hamper students' multi-digit multiplication performance. Accuracy in each of the component skills appears necessary, but not sufficient to improve composite skill performance.

Limitations

Due to confidentiality concerns, this study did not record students' demographic backgrounds, which might have influenced performance. Whether voluntary sampling was appropriate in representing the population is questionable and must be interpreted with a caution. The students were not analyzed in regards to gender which may be an issue for future studies. Lastly, the presentation of the materials (i.e., assessments) were not counter balanced which might have affected the outcomes.

CONCLUSION

This study assessed 156 fifth-graders' performance in numeral writing and component and composite multiplication facts for accuracy and fluency respectively measured by percent correct and correct digits per minute. The results demonstrated that students had achieved high levels of accuracy with single digit multiplication but slightly lower levels with multi-digit multiplication. Fluency performances for multiplication facts showed students lack competence with both component, single digit, and composite, multi-digit multiplication problems. Application, or instances where a component skill is applied to a composite skill, was revealed through the strong positive correlation of fluency with the single digit and multi-digit multiplication performances. The findings from this study systematically replicate previous application research and support the importance of fluency in elementary mathematics curricula. Increasing fluency in basic mathematic competence may well provide an alternative solution for cumulative mathematical deficits.

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